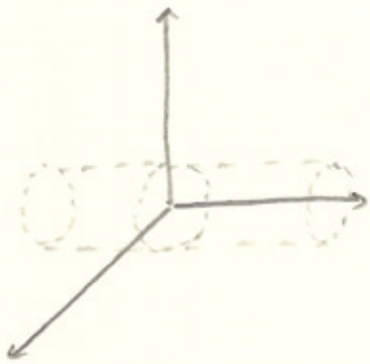
§ 13.7 CYLINDERICAL AND SPHERICAL COORDINATES.

EX: Identify the surface with the equation.

$$\rho^2(\sin^2\phi\cos^2\theta + \cos^2\phi) = 4$$

writer as

$$x^2 + z^2 = 4$$



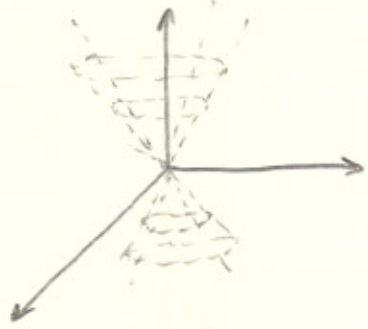
note: cylinder.

another eq.

$$\rho^2(\sin^2\phi - \cos^2\phi) = 0$$

$$r^2 - z^2 = 0$$
$$x^2 + y^2 - z^2 = 0$$

$\Big\}$ $z = \pm r$



note: top and bottom of cone.

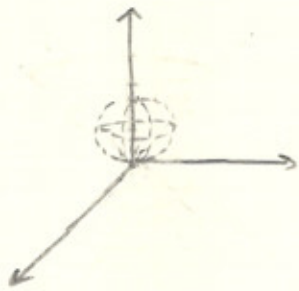
EX: Identify the surface with the eqn.

$$\rho = 2\cos\phi$$

SOL

$$\rho^2 = 2\rho\cos\phi$$

$$\rho^2 = 2z \quad \therefore z = \frac{\rho^2}{2}$$



note: $\rho = \sqrt{x^2 + y^2 + z^2}$

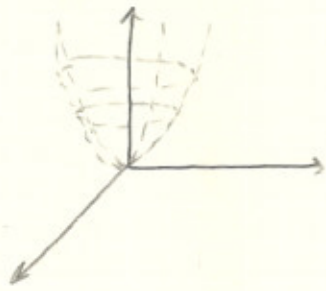
EX: what is the solid determined by

$$r^2 \le z \le 2 - r^2$$

$$x^2 + y^2 \le z \le 2 - x^2 + y^2$$

just look at

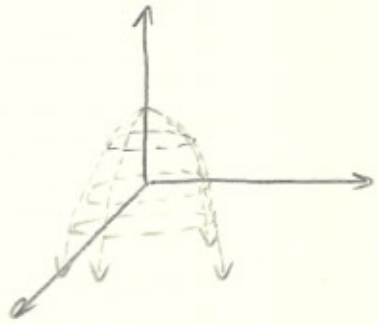
$$z = x^2 + y^2$$



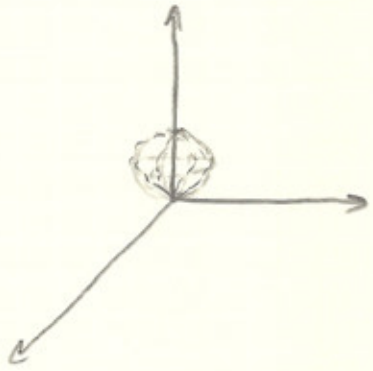
and with $z \ge x^2 + y^2$, we fill the solid.

and we look at.

$$z = 2 - x^2 + y^2$$



for $z \leq 2 - x^2 + y^2$, we fill the solid again. combining the two



note: solid (football shape

EX: write the equation of the plane containing the following lines.

$$\begin{aligned} x &= 3 + 2t \\ y &= t \\ z &= 8 - t \end{aligned} \qquad \begin{aligned} x &= s + t \\ y &= 4 - t \\ z &= 6 \end{aligned}$$

find 2 pts on line 1.

$P(3, 0, 8)$
$Q(5, 1, 7)$

find 2 pts on line 2.

$R(5, 4, 6)$
$S(6, 3, 6)$

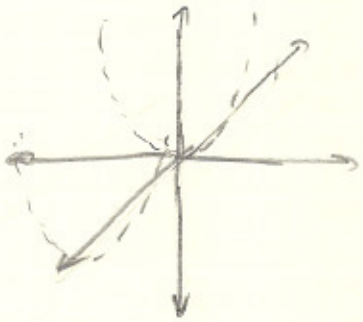
find the 2 vectors, cross product, then use one of the points to find plane

Ex. Draw the surface

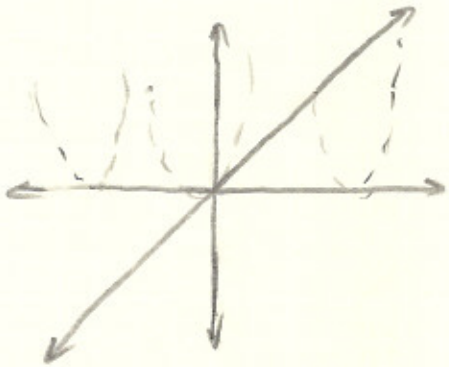
$$z = x^2 + y^2$$

Sol: we will use the method of cuts, that is to try and visualise the surface by understanding how the cross sections look like.

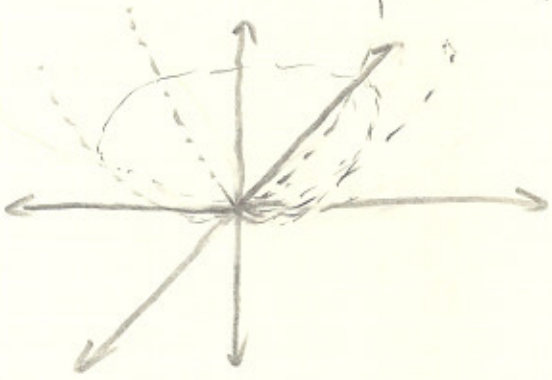
when $x = 0$ the eq becomes, $z = y^2$



when $y = 0$ the eq becomes $z = x^2$



so when you put them together,

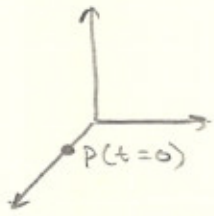


upper

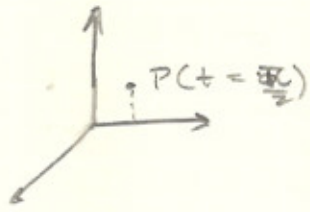
EX: What is the curve determined by

$r(t) = \langle \cos t, \sin t, t \rangle$

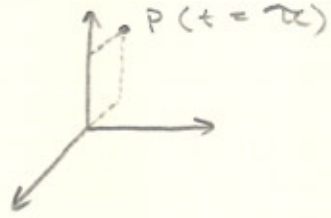
$t = 0$



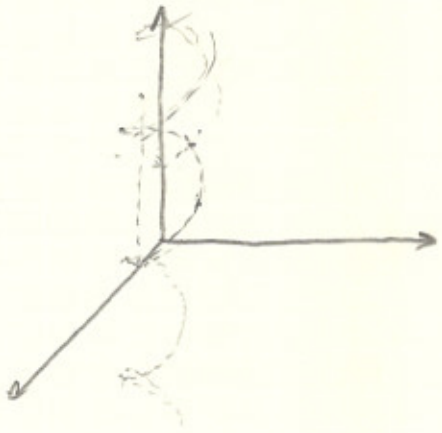
$t = \frac{\pi}{2}$



$t = \pi$

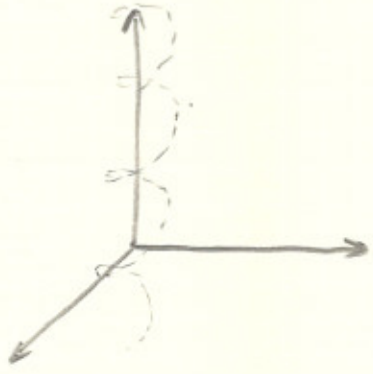


$\therefore$



note: spirl around z-

EX: $r(t) = \cos(t^2)\vec{i} + \sin(t^3)\vec{j} + t^2\vec{k}$.

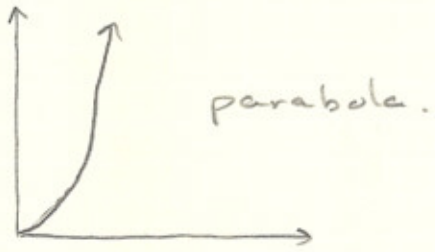


note: same shape just moving faster.

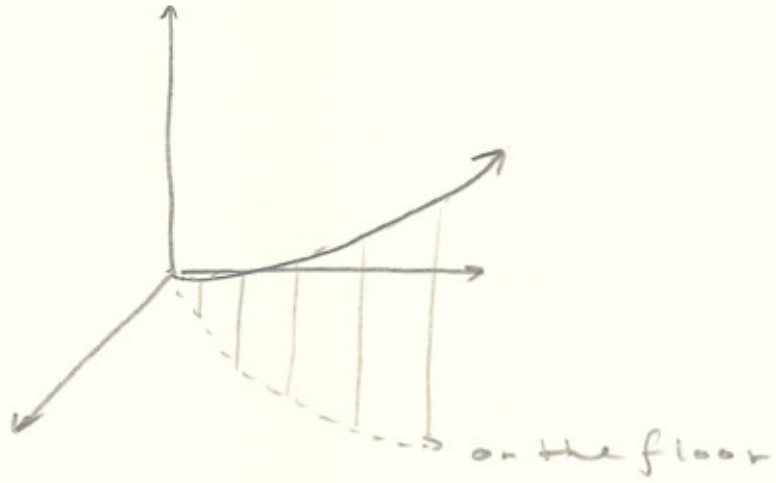
EX:

$$r(t) = t^2\vec{i} + t^4\vec{j} + t^6 \cdot \vec{k}$$

2D



3D,

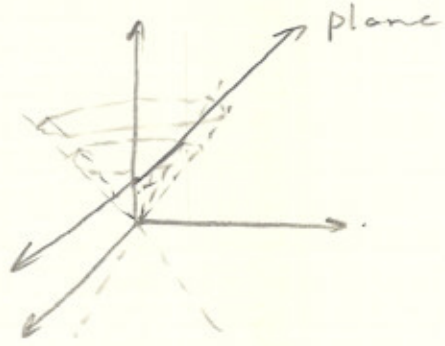


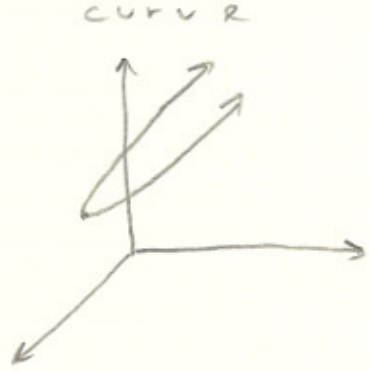
EX: find the vector function, that corisponds to the curve given by the intersection of the cone

$$z = \sqrt{x^2 + y^2}$$

and the plane

$$z = 1 + y$$





SOL:

$$\sqrt{x^2+y^2} = 1+y$$

$$x^2+y^2 = 1+2y+y^2$$

$$x^2 = 1+2y$$

$$\therefore y = \frac{1}{2}(x^2-1)$$

$$x = t$$

$$y = \frac{1}{2}(t^2-1)$$

$$z = 1+\frac{1}{2}(t^2-1)$$