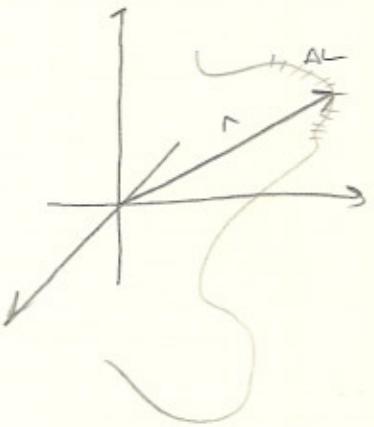
§ 17.2 INTEGRALS OVER A CURVE.



$r(x,y,z)$ or $r(x,y)$

parameterise to....

$f(x(t), y(t),)$ } density function

length $= \Sigma f(x_i, y_i)\, \Delta L$

$$\int_C f(x,y)\, ds = \int f(x(t), y(t)) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

EX:

$$\int_C (2 + x^2 y)\, ds$$ where C is a unit circle.

SOL:

$x = \cos\theta$
$y = \sin\theta$ $\quad 0 < \theta < \pi$

$$\int (2 + \cos^2\theta \sin\theta) \sqrt{(-\sin\theta)^2 + \cos^2\theta}\, d\theta$$

for 3 dimensions:

$$\int_C f(x,y,z)\,ds = \int f(x(t),y(t),z(t))\sqrt{\left(\frac{dz}{dt}\right)^2+\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$$

Unified formula for curves in 2,3 dimensions

$$\int_C f = \int f(r(t))\,|r'(t)|\,dt$$

TWO OR MORE INTERALS OVER A CURVE.

$$\int_C f(x,y)\,dx = \int f(x(t),y(t))\,x'(t)\,dt$$

$$\int_C f(x,y)\,dy = \int f(x,(t),y(t))\,y'(t)\,dt.$$

note: $$\int_C f(x,y)\,dx = -\int_{-C} f(x,y)\,dy$$