

## § 17.4 GREENS THEOREM

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

↑  
vector field

 $\mathbb{R}^3$ 

$$\begin{aligned} x &= x(t) \\ y &= y(t) \\ z &= z(t) \end{aligned}$$

} everything is in parameter

$$\mathbf{r}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}$$

$$= \int_C \mathbf{F} \cdot d\mathbf{r} = \int_{\substack{\text{domain of} \\ t}} \mathbf{F} \cdot \mathbf{r}'(t) dt$$

$$\mathbf{F}(x, y, z) = P(x, y, z)\vec{i} + Q(x, y, z)\vec{j} + R(x, y, z)\vec{k}$$

$$= \int_{\substack{\text{domain of} \\ t}} (P(x, y, z)x'(t) + Q(x, y, z)y'(t) + R(x, y, z)z'(t)) dt$$

$$= \int_C Pdx + Qdy + Rdz$$

GREEN'S THEOREM: let  $C$  be a closed domain in  $\mathbb{R}^2$  which encloses a domain is zero.

$$\int_C P dx + Q dy = \iint_D \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$$



note: the curve  $C$  is traced counterclockwise notation.

$$\oint P dx + Q dy,$$

Green's Theorem is similar to the FTC for a double integral  $C$ .