

§ 17.1 VECTOR FUNCTIONS.

vector fields in \mathbb{R}^3 is a function.

$$f(x, y, z)$$

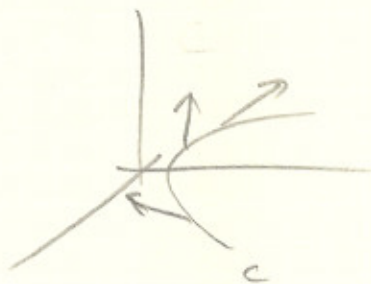
$$= P(x, y, z)\vec{i} + Q(x, y, z)\vec{j} + R(x, y, z)\vec{k}$$

for every point in space we attach a vector,

§ 17.2 LINE INTEGRALS OF VECTOR FIELD

Q: suppose that an object is measured along the curve C , such that a different force F is applied to every moment. find the total work done.

remark: In the case that F is an example of a vector field.



$$C: \begin{cases} x(t) \\ y(t) \\ z(t) \end{cases}$$

$$\int_C f \, ds = \int_a^b f(r(t))$$

DEF:

$$\int_C F \cdot dr = \int_{\text{domain of } t} f(r(t)) \cdot r'(t) dt$$

dot product

EX: find

$$\int_C F dr$$

$$F(x, y, z) = xy\vec{i} + yz\vec{j} + zx\vec{k}$$

$$C = \begin{cases} x = t \\ y = t^2 \\ z = t^3 \end{cases}$$

$$\therefore r(t) = t\vec{i} + t^2\vec{j} + t^3\vec{k}$$

$$r'(t) = \vec{i} + 2t\vec{j} + 3t^2\vec{k}$$

$$\int_C F \cdot dr = \int_0^1 F(t, t^2, t^3) \cdot \vec{i} + 2t\vec{j} + 3t^2\vec{k} dt$$

$$= \int_0^1 (t^3 + 2t^6 + 3t^6) dt$$

$$= \left[\frac{t^4}{4} + \frac{2}{7}t^7 + \frac{3}{7}t^7 \right]_0^1$$

$$= \frac{1}{4} + \frac{2}{7} + \frac{3}{7} = \frac{27}{28}$$