Ex: find the volume of the solid E that has — the cylinder $x^2+y^2=1$, below the sphere $x^2+y^2+z^2=1$ and above the plane $z=-2$

SOL: $$\text{Vol}\,E = \iiint 1\,dV = \iint\left(\int_{-2}^{-\sqrt{1-x^2-y^2}} 1\,dz\right)dx\,dy$$

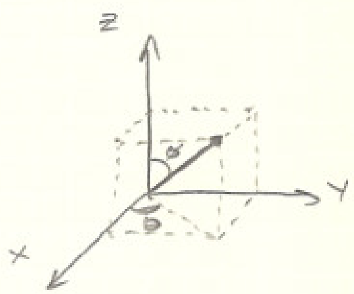
$$= \iint\left(-\sqrt{1-x^2-y^2}+2\right)dx\,dy$$

$$= \int_0^{2\pi}\int_0^1\left(-\sqrt{1-r^2}+2\right)\cdot r\,dr\,d\theta$$

$$= \int_0^{2\pi}\left(\frac{1}{3}(1-r^2)^{3/2}+\cancel{2}\frac{r^2}{\cancel{2}}\right)\Big|_0^1 d\theta$$

$$= \int_0^{2\pi}\left(1-\tfrac{1}{3}\right)d\theta = \frac{2}{3}2\pi = \frac{4}{3}\pi$$

§16.8 TRIPLE INTEGRALS USING SPHERICAL COORDINATES.



$x = (\rho\sin\phi)\cos\theta$

$y = (\rho\sin\theta)\sin\theta$

$z = \rho\cos\phi$

if the solid $E = (\rho, \phi, \theta)$ then

$$\iiint_E F(x,y,z)\,dV$$

$$\int_b^a \int_d^c \int_f^e f(\rho\sin\phi\cos\theta, \rho\sin\phi\sin\theta, \rho\cos\phi)\rho^2\sin\phi\, d\rho\, d\phi\, d\theta$$

Ex:

$$\iiint e^{(x^2+y^2+z^2)^{3/2}}\, dV$$

$$\iiint e^{(\rho^2)^{3/2}} \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$$

let $u = \rho^3$

$du = 3\rho^2\, d\rho$

$$\iiint_0^1 e^u \frac{1}{3}\sin\phi\, du\, d\phi\, d\theta$$

$$\iint($$

$$\int_0^{2\pi}\left(\frac{e}{3} - \frac{1}{3} - \left(-\frac{e}{3} + \frac{1}{3}\right)\right) d\theta$$

$$\int_0^{2\pi}\left(\frac{2e}{3} - \frac{2}{3}\right) d\theta = \frac{2e-2}{3} \cdot 2\pi$$