from last day.

what is the equation of a line that passes thru a point $P(x_0, y_0, z_0)$ and travels in the same direction as vector $v = \langle a, b, c \rangle$

the line is formed from the points

$$Q\,(x_0 + ta,\ y_0 + tb,\ z_0 + tc)$$

where t is and $\mathbb{R}$

solving for t. and then forget about it.

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

<u>EX</u>: what is the equation of the line that passes through. P & Q

$$P = (-2, 0, 1)$$
$$Q = (3, -4, 1)$$

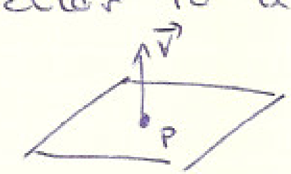
$$\overrightarrow{PQ} = \langle 3-(-2), -4-0, 1-1 \rangle$$
$$= \langle 5, -4, 0 \rangle$$

now the equation becomes.

$$\frac{x+2}{5} = \frac{y}{-4} = \cancel{\frac{z-1}{0}} \quad 1$$

PLANES.

it is easy to write the equation of a plane that cotains a point P, and which is perpendicular to a given vector $v = (a, b, c)$



In such a situation, for any point $Q(x, y, z)$ in the plane we have.

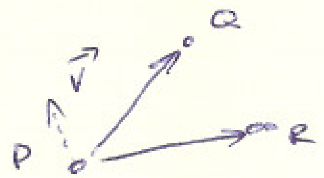
$$V \cdot \overrightarrow{PQ} = 0$$

$$\langle a, b, c \rangle \cdot \langle x - x_0, y - y_0, z - z_0 \rangle = 0$$

$$ax - ax_0 + by - by_0 + cz - cz_0 = 0$$

and this would be the equation of the plane.

EX: given 3 points $P(-2, 1, 0)$, $Q(3, 4, -1)$ and $R(-1, -1, 2)$. They will always create a ~~plane~~ plane as long as they are not on the same line

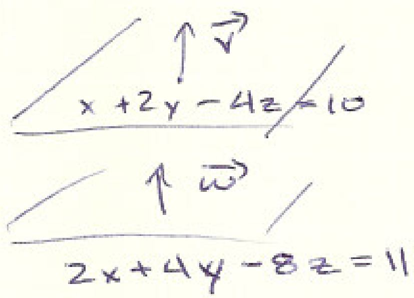


$$\vec{V} = \overrightarrow{PR} \times \overrightarrow{PQ} = \begin{vmatrix} i & j & k \\ 1 & -2 & -2 \\ 5 & 3 & -1 \end{vmatrix} = 8i - 9j + 13k$$

now the equation of the plane

$$8(x+2) - 9(y-1) + 13(z-0) = 0$$

EX: determine wether the planes $x + 2y - 4z = 10$ and $2x + 4y - 8z = 11$ are parallel.



note: given a plane with the equation

$$ax + by + \cancel{z}z - d = 0$$

then the perpendicular vector is

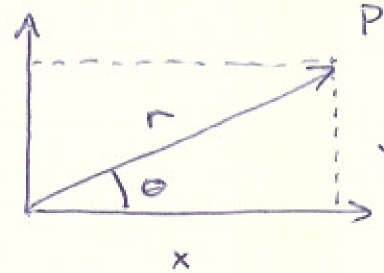
$$\vec{v} = \langle 1, 2, -4 \rangle$$
$$\vec{w} = \langle 2, 4, -8 \rangle$$

observe that $w = 2v$, they have the same direction, so the planes are parallel.

§13.7 Cylindrical and spherical coordinates.

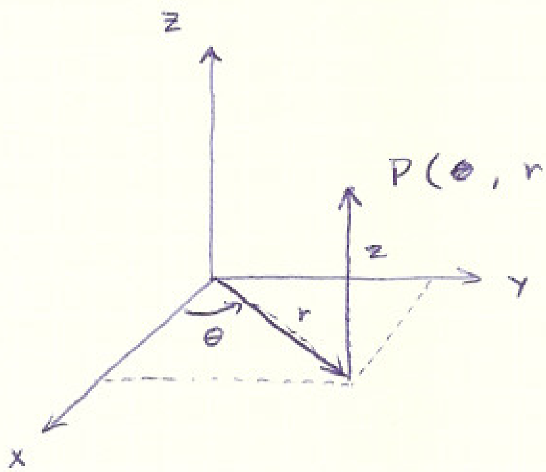
Review of Polar coordinates (2D)

$P(x, y) = P(r, \theta)$



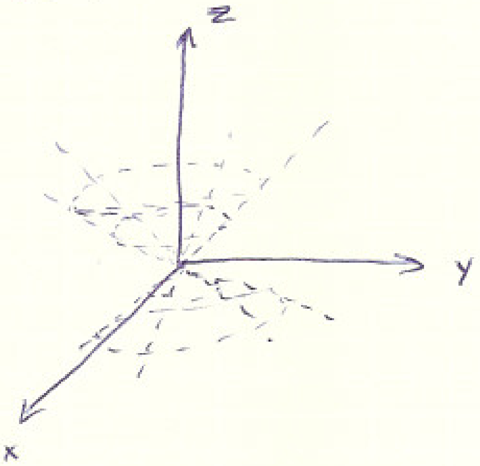
note: θ is in radians.

CYLINDERICAL COORDINATES



EX: what is the surface given by the equation

$z = r$

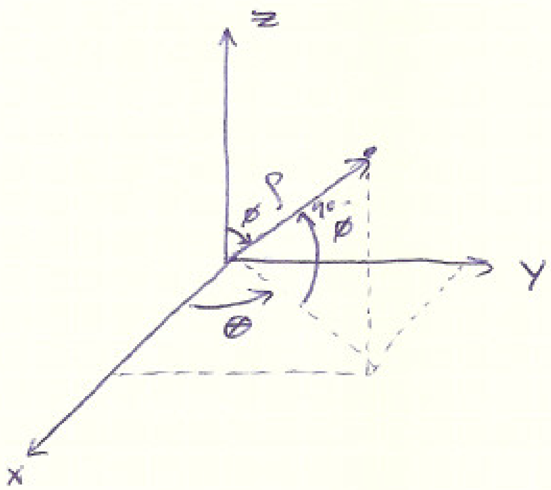


note: cone.

note: no negative radius.

observe: $z = r = \sqrt{x^2 + y^2}$

SPHEREICAL COORDINATES.



$P(\rho, \theta, \phi)$

length angles.

note: $0 \leq \theta \leq 2\pi$

$0 \leq \phi \leq \pi$

changing coordinates.

$z = \rho \cos \phi$

$$x = \rho \sin\phi \cos\theta$$
$$y = \rho \sin\phi \sin\theta$$

EX: what is the solid satisfying

$$0 \le \rho \le 2$$
$$0 \le \phi \le \frac{\pi}{2}$$
$$0 \le \theta \le \frac{\pi}{4}$$