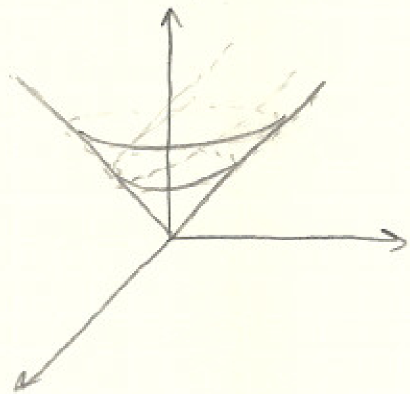
COVERS EVERYTHING UP TIL §15.1

Ex: curve obtained by intersecting

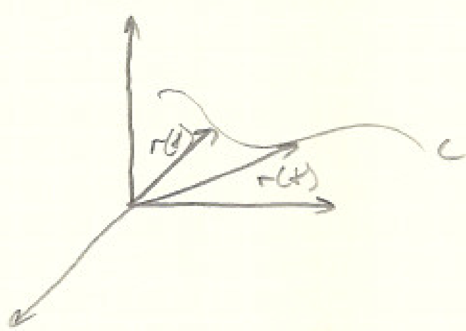
$z = \sqrt{x^2+y^2}$ (cone) with $x = 1 + z$



VECTOR FUNCTIONS.

continued from last day

$$r(t) = \langle f(t), g(t), h(t) \rangle = f(t)\,i + g(t)\,j + h(t)\,k$$



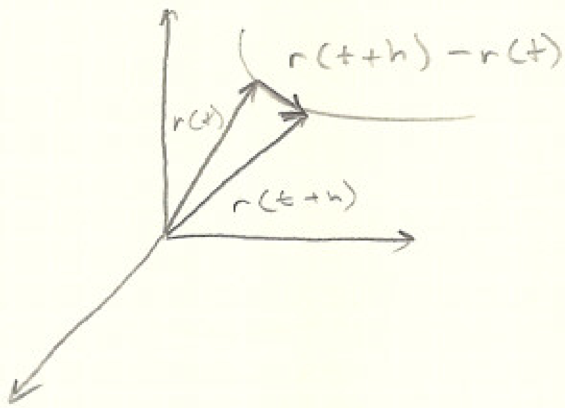
§14.2 DERIVATIVES & INTEGRAL OF VECTOR FUNCTION

$$\lim_{h \to 0} \frac{r(t+h) - r(t)}{h} = r'(t)$$

$r(t) = \langle f(t), g(t), h(t) \rangle$

$r'(t) = \langle f'(t), g'(t), h'(t) \rangle$

(when all derivatives



geometric interputation: For a vector function $r(t)$, the derivative $r'(t)$ is a vector that gives the direction of the tangent line to the curve determined by this vector function.

Ex. for a helix

$$x = 2\cos t$$
$$y = \sin t$$
$$z = t$$

find the equation of a tangent line to this helix, at poin $(0, 1, \frac{\pi}{2})$

SOL:

$$\left.\begin{array}{l} x' = -2\sin t \\ y' = \cos t \\ z' = 1 \end{array}\right\} r'(t)$$

when $t = \frac{\pi}{2}$ $\therefore$ $r'(\frac{\pi}{2}) = \langle -2, 0, 1 \rangle$

$\therefore$ eq of line

$$x = -2t.$$
$$y = 1$$
$$z = \frac{\pi}{2} + t$$

TANGENT VECTOR

a vector with length 1 tangent to the curve determined by $r(t)$

$$T(t) = \frac{r'(t)}{|r'(t)|}$$

NORMAL VECTOR.

a vector of length 1. The normal vector is perpendicular to $T(t)$

$$N(t) = \frac{T'(t)}{|T'(t)|}$$

DERIVATION RULES OF VECTORS

$$\frac{d}{dt}(\overrightarrow{u(t)} + \overrightarrow{v(t)}) = \frac{d}{dt}u(t) + \frac{d}{dt}v(t)$$

$$\frac{d}{dt}[u(t) \cdot v(t)] = \frac{du(t)}{dt} \cdot v(t) + \frac{dv(t)}{dt} \cdot u(t)$$

$$\frac{d}{dt}[u(t) \times v(t)] = \frac{du(t)}{dt} \times v(t) + \frac{dv(t)}{dt} \times u(t)$$

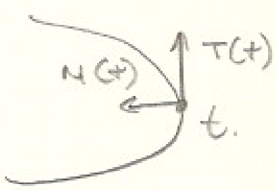
PROOF FOR NORMAL VECTOR

$$T(t) \cdot T(t) = 1$$

taking derivative

$$\frac{d}{dt}[T(t) \cdot T(t)] = T'(t) \cdot T(t) + T'(t) \cdot T(t) = 0$$

$$T'(t) \cdot T(t) = 0 \longrightarrow N(t) \cdot T(t) = 0$$



$\therefore \quad N(t) = T'(t)$

$\therefore \quad N(t) = \dfrac{T'(t)}{|T'(t)|}$

BINOMIAL VECTOR

a vector of length 1 which is perpendicular to both T(t) and N(t)

$$B(t) = T(t) \times N(t)$$

"oscilating plane" → is a plane determined by T(t) and N(t), the plane that comes closest to containing the part of the curve.

"normal line" → is a plane determined by N(t) and B(t)

EX: Find the equation of the oscilating and normal plane for the helix.

$$r(t) = \cos t \cdot \vec{i} + \sin t\,\vec{j} + t \cdot \vec{k}$$

$$P(1, 0, 2\pi)$$

SOL: Pt P corispond with $t = 2\pi$

$$r'(t) = -\sin t\,\vec{i} + \cos t\,\vec{j} + \vec{k}$$

$$|r'(t)| = \sqrt{(-\sin^2(1)) + \cos(0)^2 + 1^2} = \sqrt{2}$$

$$T(t) = \frac{r'(t)}{|r'(t)|} = \left\langle -\frac{\sin t}{\sqrt{2}}, +\frac{\cos t}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$$

$$T'(t) = \left\langle -\frac{\cos t}{\sqrt{2}}, -\frac{\sin t}{\sqrt{2}}, 0 \right\rangle$$

$$|T'(t)| = \sqrt{\left(\frac{\cos^2 t}{\sqrt{2}}\right) + \left(\frac{\sin^2 t}{\sqrt{2}}\right)} = \frac{1}{\sqrt{2}}$$

$$N(t) = \frac{T'(t)}{|T'(t)|} = \langle -\cos t, -\sin t, 0 \rangle$$

we need $T(2\pi), N(2\pi), B(2\pi)$

$$T(2\pi) = \langle 0, 1, 1 \rangle$$

$$N(2\pi) = \langle -1, 0, 0 \rangle$$

$$B(2\pi) = \langle 0, -\tfrac{1}{\sqrt{2}}, \tfrac{1}{\sqrt{2}} \rangle$$

oscilating plane $<$ point $(1, 0, 2\pi)$
vector perp to plane $(B(t))$

.... $0(x-1) + \frac{1}{\sqrt{2}}(y-0) + \frac{1}{\sqrt{2}}(z-2\pi) = 0$

$$-\frac{1}{\sqrt{2}}y + \frac{1}{\sqrt{2}}z - \frac{2}{\sqrt{2}}\pi = 0$$

normal plane $<$ point $(1, 0, 2\pi)$
vector per to plane $T(t)$

.... $0(x-1) + 1(y-0) + 1(z-2\pi) = 0$

$$y + z - 2\pi = 0$$