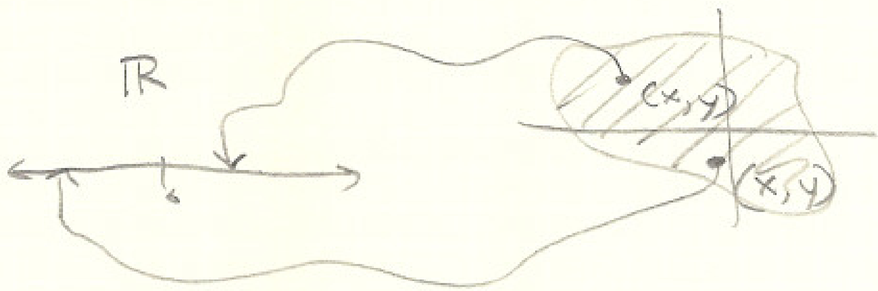
§15.1 FUNCTIONS OF SEVERAL VARIABLES

functions in 2 variables

$f(x,y) =$ a # depending on x & y

$f: D \longrightarrow \mathbb{R}$

pairs of numbers



Ex: draw the regions in the 2d plane that represent the domains of the following functions.

1) $f(x,y) = \ln(x - y + 5)$

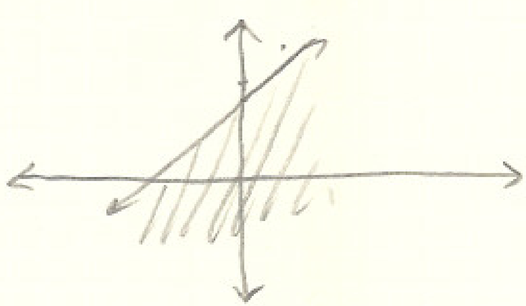
2) $g(x,y) = \sqrt{y^2 - x}$

3) $h(x,y) = \dfrac{\sqrt{4 - x^2 - y^2}}{\ln x}$

SOL:

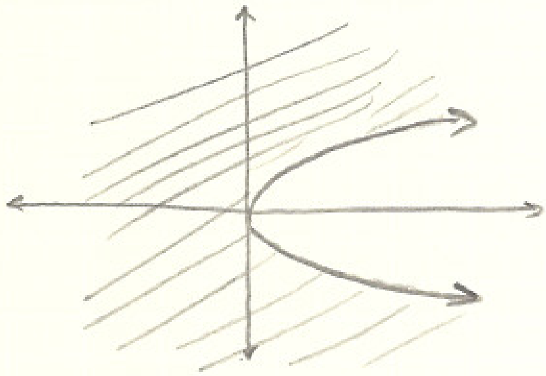
$D(x,y)$ such that $x - y + 5 > 0$

$x + 5 > y$



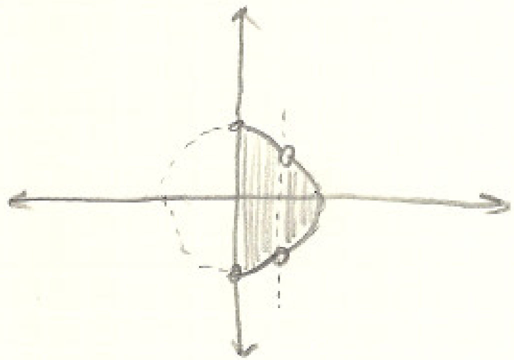
2.)

$$y^2 - x \geq 0$$
$$y^2 \geq x$$



3)

$$x \neq 1 \ , \ x > 0$$
$$4 - x^2 - y^2 \geq 0$$
$$4 \geq x^2 + y^2$$



GRAPH OF A FUNCTION IN 2 VARIABLES.

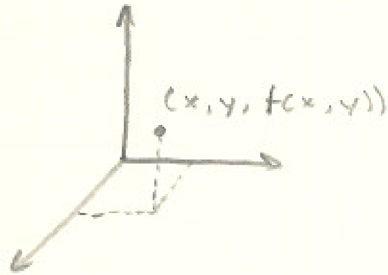
from calc 2

$$f(\mathbb{R}) \longrightarrow \mathbb{R}$$

now for $f: D \longrightarrow \mathbb{R}$

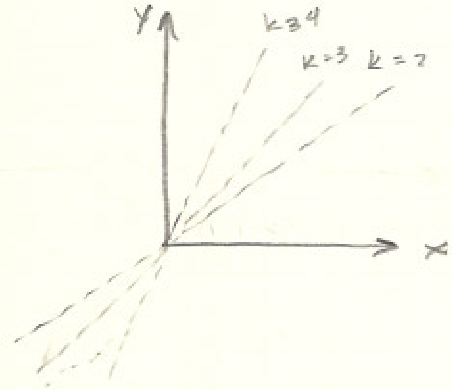
region in 2 dimensional plane

think $f(x,y) = z$.



Ex: what is the graph of $f(x,y) = e^{y/x}$

$x \neq 0$



§15.2 LIMITS OF FUNCTIONS IN 2 VARIABLES.

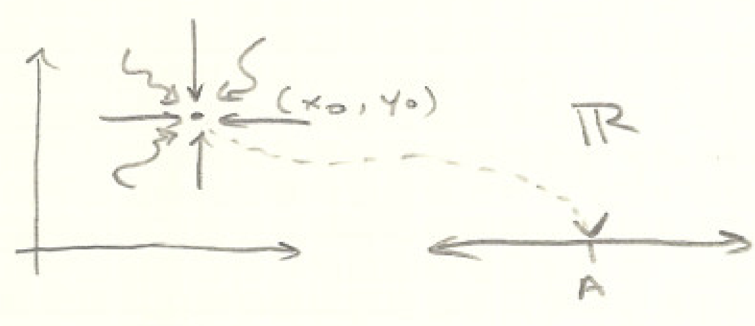
from Calc I

$$\lim_{x \to x_0} f(x) = A$$

means that as x appoaches x_0, then $f(x)$ approaches A.

Now

$$\lim_{(x,y) \to (x_0, y_0)} f(x,y) = A$$



Ex: show that

$$\lim_{(x,y) \to (0,0)} \frac{x^2 - y^2}{x^2 + y^2} = \text{doesn't exist.}$$

from approaching this point from multiple directions we can see that this point has different limits