§17.7 SURFACE INTEGRAL (OF VECTOR FUNCTIONS)

S: surface in 3 dimensions (we need 2 para)

$$S: \begin{cases} x(u,v) \\ y(u,v) \\ z(u,v) \end{cases}$$

$$r(u,v) = x(u,v)\vec{i} + y(u,v)\vec{j} + z(u,v)\vec{k}$$

$$\iint_S f(x,y,z)\,dS = \iint_{\substack{\text{domain} \\ \text{of } u,v}} f(x(u,v)\,y(u,v)\,z(u,v))\,|r_u \times r_v|\,dA$$

NOW GIVEN VECTOR FEILD.

$$\iint_S F \cdot ds = \iint_{\substack{\text{domain} \\ \text{of } u,v}} F(x(u,v)\,y(u,v)\,z(u,v)) \cdot (\vec{r}_u \times \vec{r}_v)\,dA.$$

dot product.

<u>note</u>: both $r_u \times r_v$ are in the tangent plane of the surface.

<u>EX</u>: $\iint_S F \cdot ds$, where $F(x,y,z) = \vec{i} + y\vec{j} + 5z\vec{k}$

S: is part of the cylinder $x^2+y^2=4$ bounded by $z=0$, $x+z=2$

$x = 2\cos\theta$
$y = 2\sin\theta$
$z = z$

$$r(\theta, z) = 2\cos\theta\,\vec{i} + 2\sin\theta\,\vec{j} + z\vec{k}$$

$$D: \begin{cases} 0 \le \theta \le 2\pi \\ 0 \le z \le 2 - \underbrace{2\cos\theta}_{x} \end{cases}$$

$$r_\theta = -2\sin\theta\,\vec{i} + 2\cos\theta\,\vec{j} + 0\cdot\vec{k}$$
$$r_z = 0\cdot\vec{i} + 0\cdot\vec{j} + 1\cdot\vec{k}$$

$$r_z \times r_\theta = \begin{vmatrix} i & j & k \\ -2\sin\theta & 2\cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= 2\cos\theta\,\vec{i} + 2\sin\theta\,\vec{j} + 0\cdot\vec{k}$$

now just compute

$$\int_0^{2\pi}\int_0^{2-2\cos\theta} F(z,\theta)\cdot(r_z \times r_\theta)\,dz\,d\theta$$