

§16.9 CHANGE OF VARIABLES IN MULTIPLE INTEGRATION

① double integral

main idea: suppose we have to compute $\iint_R f(x,y) dA$, where R looks complicated

suppose that we can write

$$x = x(u,v)$$

$$y = y(u,v)$$

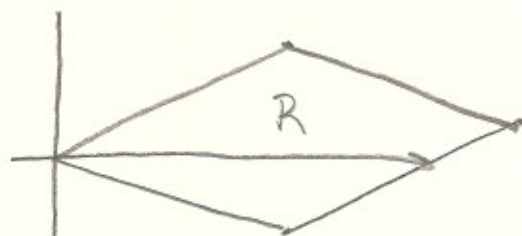
in terms of some other system of coordinates u,v in such a way that the domain R (in x,y coordinates) is transformed into a simpler domain S (in u,v coordinates)

then we can compute the initial double integral in (x,y) in terms of u,v ,

$$\iint_R f(x,y) dA = \iint_S f(x(u,v), y(u,v)) \left| \det \begin{bmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{bmatrix} \right| dA$$

EX:

$$\iint_R y dA$$



↑
absolute
value.

$$\begin{aligned} x &= 2u+3 \\ y &= u-v \end{aligned}$$

$$\therefore u = \frac{x+3y}{5}$$

$$v = \frac{x-2y}{5}$$

(x,y)

(u,v)

$(0,0)$

$(0,0)$

$(6,3)$

$(3,0)$

$(12,1)$

$(3,2)$

$(6,-2)$

$(0,2)$