§16.4 DOUBLE INTEGRALS USING SPHERICAL COORDINATES.

$$D = \{(r,\theta) : \begin{matrix} 0 \le r \le 1 \\ 0 \le \theta \le \pi/4 \end{matrix}$$

polar rectangle is a domain in $\mathbb{R}^2$, which can be written as

$$D = \left\{(r,\theta) : \begin{matrix} a \le r \le b \\ c \le \theta \le d \end{matrix}\right\}$$

Theorem:

$$\iint_D = \int_c^d \left(\int_a^b f(r\cos\theta, r\sin\theta) r\, dr \right) d\theta$$

Ex. $$\iint_D \arctan\left(\frac{y}{x}\right) dA = \int_0^{\pi/4} \left(\int_1^2 \arctan\left(\frac{r\sin\theta}{r\cos\theta}\right) r\, dr \right) d\theta$$

$$\int_0^{\pi/4} \left(\int_1^2 \underbrace{\tan^{-1}(\tan\theta)}_{\theta} r\, dr \right) d\theta = \int_0^{\pi/4} \left(\theta \frac{r^2}{2} \Big|_1^2 \right)$$

$$\int_0^{\frac{\pi}{4}} \frac{3}{2}\theta = \frac{3}{4}\theta^2 \Big|_0^{\pi/4} = \frac{3\pi}{64}$$

§ 16.6 TRIPLE INTEGRALS.

$f(x,y,z)$

$f : E \to \mathbb{R}$

$$\iiint_E f(x,y,z)\,dV = \iint_D \left(\int_{f(x,y)}^{g(x,y)} F(x,y,z)\,dz \right) dA$$