from last day

EX:

$$\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^4} = \text{does not exist.}$$

approaching horizontal.

$$\lim_{(x,0)\to(0,0)} \frac{x\cdot 0}{x^2+0} = 0$$

approaching vertical

$$\lim_{(0,y)\to(0,0)} \frac{0\cdot y^2}{0+y^4} = 0$$

$x = y^2$

$$\lim_{(y^2,y)\to(0,0)} \frac{y^4}{2y^4} = \frac{1}{2}$$

$\therefore$ limit does not exist.

EX: Show that.

$$\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^2}$$

SOL: we will use squize theorem.

$$0 \le \left|\frac{xy^2}{x^2+y^2}\right| = |x|\underbrace{\frac{y^2}{x^2+y^2}}_{<1} \le |x|$$

§15.3 PARTIAL DERIVATIVES

given a function in 2 variables

$f(x,y)$

we can define the partial derivatives as follows:

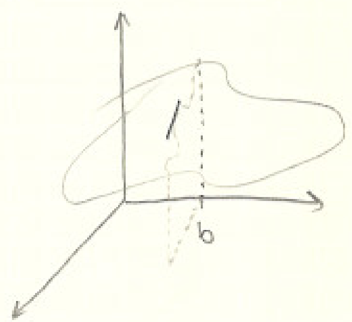
$$\frac{\partial f}{\partial x} = \frac{\partial z}{\partial x} = f_x'(x,y)$$

thinking of y as constant, then take with respect to x.

remark: of multiple orders.

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

geometric interpritation



§15.5 CHAIN RULE.

1# given $f(x,y)$, suppose $x=g(t)$, $y=h(t)$, then f becomes a function of t.

$$\frac{d}{dt} f(g(t), h(t)) = \frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dt}$$

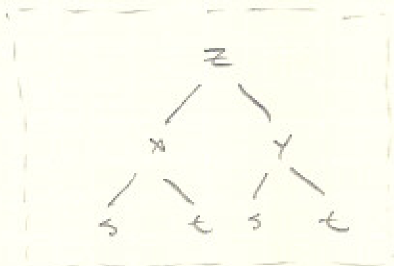
huh?

chain rule # 2

given $f(x, y)$, suppose

$$x = g(s, t)$$
$$y = h(s, t)$$

$$\therefore \quad f(g(s,t), h(s,t))$$



$$\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial s}$$

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial t}$$

Ex:

$$z = x^2 + xy + y^2 \qquad \begin{aligned} x &= s + t \\ y &= st \end{aligned}$$

$$\frac{\partial z}{\partial s} = [2x + y][1] + [x + 2y][t]$$

$$\frac{\partial z}{\partial t} = [2x + y][1] + [x + 2y][s]$$

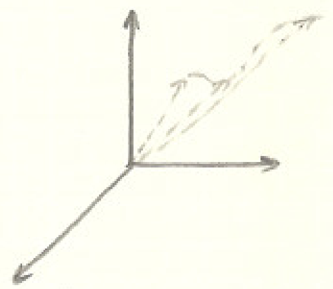
§ 14.1 VECTOR FUNCTIONS.

until now: we had functions $f(\mathbb{R}) = \mathbb{R}$

now: we will discuss functions that attach a $\mathbb{R}$ t to a vector

$$r(t) = \vec{v}$$

Think of t (time) being associated to a specific vector. (3D)



EX:

$$r(t) = \langle \sqrt{t-1}, \ln(1+t^2), \sqrt{5-t} \rangle$$

domain of $r(t) = [1, 5]$

note: A continous vector function $r(t) = \langle f(t), g(t), h(t) \rangle$ determines a curve C which is traced out by the tip of the moving vector $r(t)$.

Eq of the curve

$$x = f(t)$$
$$y = g(t)$$
$$z = h(t)$$