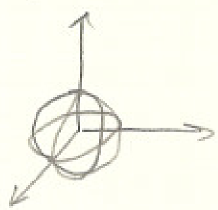
§17.6 SURFACE AREA.

In general

$S = \{(x,y,z)$ such that $\begin{array}{l} x = x(u,v) \\ y = y(u,v) \\ z = z(u,v) \end{array}$

note: with u, v in the same domain.

Ex: sphere with radius 1.



$x = \rho \sin\phi \cos\theta$

$y = \rho \sin\phi \sin\theta$

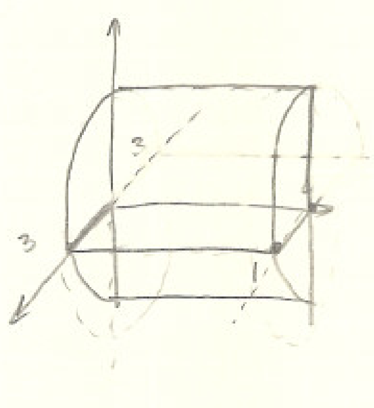
$z = \rho \cos\phi$

$\rho = 1$

$0 < \phi < \pi$

$0 < \theta < 2\pi$

Ex: parameterise the surface of cylinder $x^2+z^2=9$ enclosed by the planes. $y=0$, $y=4$, $x=0$



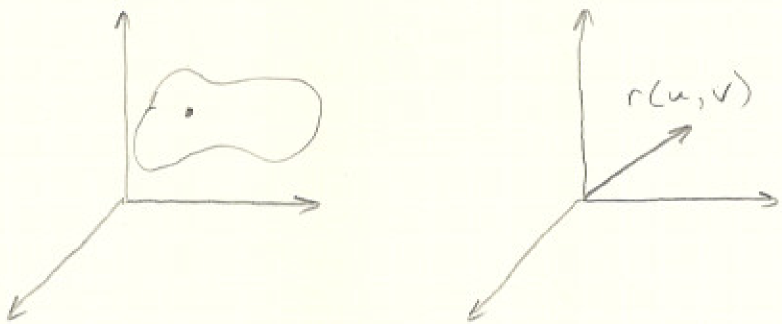
$S = \{(x,y,z) \Rightarrow \begin{array}{l} x = 3\cos\theta \\ y = y \\ z = 3\sin\theta \end{array}$

$0 < y < 4$

$\frac{-\pi}{2} < \theta < \frac{\pi}{2}$

TANGENT PLANES TO A SURFACE.

$$S = \{ (x, y, z) \qquad \begin{aligned} x &= x(u,v) \\ y &= y(u,v) \\ z &= z(u,v) \end{aligned}$$



find the value tangent to the plane.

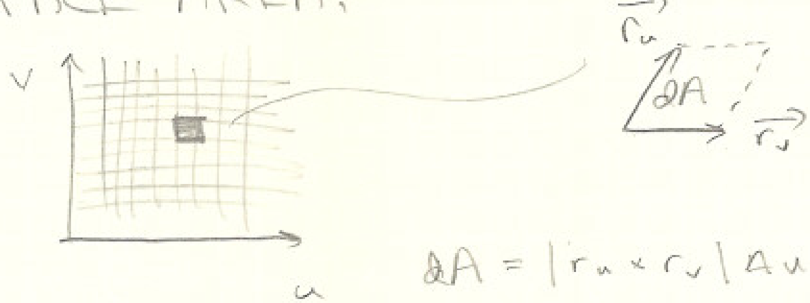
$$\vec{r}(u,v) = x(u,v)\,\vec{i} + y(u,v)\,\vec{j} + z(u,v)\,\vec{k}$$

$$\vec{r_u} = \frac{\partial x}{\partial u}\vec{i} + \frac{\partial y}{\partial u}\vec{j} + \frac{\partial z}{\partial u}\vec{k}$$

$$\vec{r_v} = \frac{\partial x}{\partial v}\vec{i} + \frac{\partial y}{\partial v}\vec{j} + \frac{\partial z}{\partial v}\vec{k}$$

Then $\vec{r_u} \times \vec{r_v}$ is a vector perpendicular to plane of the surface a point (u,v)

SURFACE AREA.



$$\partial A = |r_u \times r_v| \,\Delta u \Delta v$$

$$A(S) = \iint |\vec{r_u} \times \vec{r_v}| \, du\, dv.$$