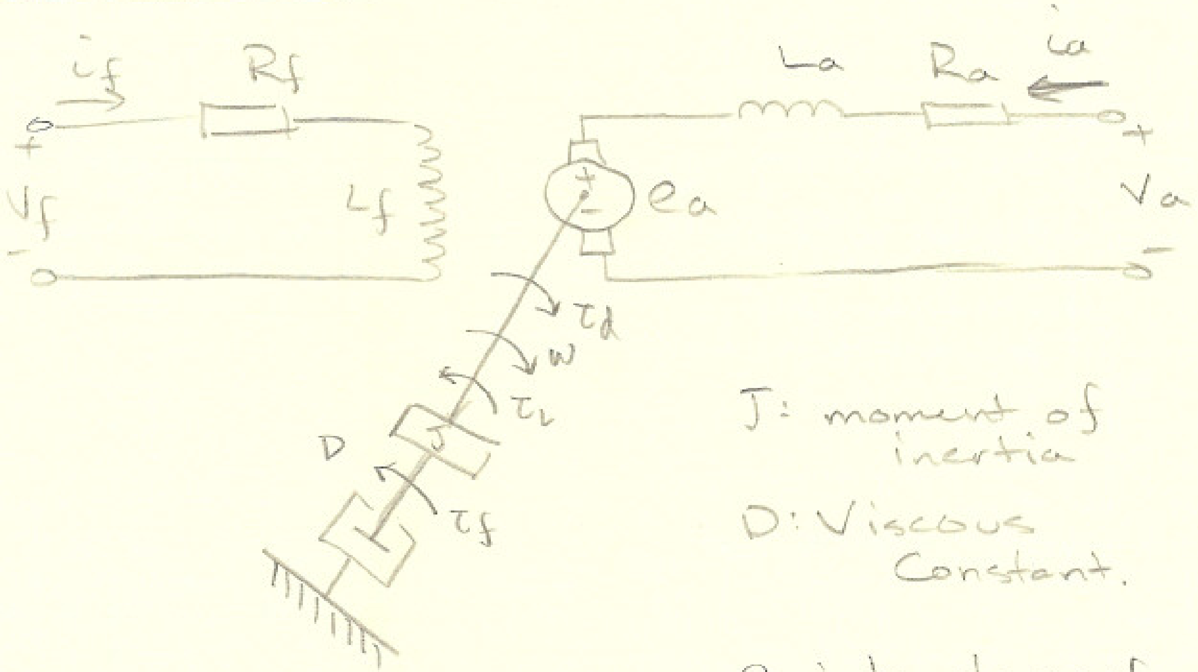
Midterm Oct 30th Ch 1 → 3

DC Motor Dynamics



J: moment of inertia

D: Viscous Constant.

e_a: back emf.

$$\tau_f = D\omega$$

By KVL

$$V_f = R_f i_f + L_f \frac{di_f}{dt}$$

$$V_a = R_a i_a + L_a \frac{di_a}{dt} + e_a$$

By Newtons Law

$$\tau_d - \tau_L - \tau_f = J\frac{d\omega}{dt}$$

$$e_a = K_e i_f \omega$$ Voltage constant.

$$\tau_d = k_\tau i_f i_a$$ Tourque constant

$$\left.\begin{aligned} i_f &= i_f(t) \\ i_a &= i_a(t) \\ \omega &= \omega(t) \\ v_f &= v_f(t) \\ v_a &= v_a(t) \\ \tau_L &= \tau_L(t) \end{aligned}\right\} \text{functions of time.}$$

Steady State Operation.

$$\frac{di_f}{dt} = 0 \qquad \frac{di_a}{dt} = 0 \qquad \frac{d\omega}{dt} = 0$$

$$\therefore \quad v_f = R_f i_f$$

$$v_a = R_a i_a + e_a$$

$$\tau_d - \tau_L - \tau_f = 0$$

$$e_a = K_e i_f \omega$$

$$\tau_d = K_\tau i_f i_a$$

$$\frac{di_f(t)}{dt} = -\frac{R_f}{L_f} i_f(t) + \frac{V_f(t)}{L_f}$$

$$\frac{di_a(t)}{dt} = -\frac{R_a}{L_a} i_a(t) - \frac{K_e}{L_a} i_f(t)\omega(t) + \frac{V_a}{L}$$

$$\frac{d\omega(t)}{dt} = \frac{K_e}{J} i_f(t) i_a(t) - \frac{T_L(t)}{J} - \frac{D}{J}\omega(t)$$

$\mathcal{L} \rightarrow$ yeilds.

$$sI_f(s) - i_f(0) = \frac{R_f}{L_f} I_f(s) + \frac{1}{L_f} V_f(s)$$

$$(L_f s + R_f) I_f(s) = V_f(s) + L_f i_f(0)$$

$$I_f(s) = \frac{V_f(s) + L_f i_f(0)}{L_f s + R_f}$$

Armature-Controlled DC motor:

Assume $i_f(t) = i_f$ (constant)

$$I_f = \frac{V_f}{R_f}$$

$$\therefore \quad L_a \frac{di_a(t)}{dt} = -R_a i_a(t) - K_e I_f \omega(t) + v_a(t)$$

$$L_a s I_a(s) - L_a i_a(0) = -R_a I_a(s) - K_e I_f \omega(s) + V_a(s)$$

$$I_a(s) = \frac{[V_a(s) + L_a i_a(0)] + K_e I_f \omega(s)}{sL_a + R_a}$$

$$Js\omega(s) - J\omega(0) = K_e I_f I_a(s) - \tau_L(s) - D\omega(s)$$

$$(Js + D)\omega(s) = K_e I_f I_a(s) + J\omega(0) - \tau_L(s)$$

$$(Js + D)\omega(s) = K_e I_f \frac{[V_a(s) + L_a i_a(0)] - K_e I_f \omega(s)}{L_a s + R_a} + J\omega(0) - \tau_L(s)$$

$$(L_a s + R_a)(Js + D)\,\omega(s) = K_e I_f\left[V_a(s) + L_a i_a(0)\right]$$

$$-(K_e I_f)^2\,\omega(s) + (L_a s + R_a)(J\omega(0) + \tau_L(s))$$

$$\omega(s) = \frac{K_e I_f\left[V_a(s) + L_a i_a(0)\right] + (L_a s + R_a)(+J\omega(0) + \tau_L(s))}{(L_a s + R_a)(Js + D) + (K_e I_f)^2}$$

See Lecture Notes for more exact derivativation.