EX:

$V_f = 240\ V$
$V_a = 240\ V$
$P = 12\ HP$
$R_a = 0.28\ \Omega$
$L_a = 2.81\ mH$
$R_f = 320\ \Omega$
$L_f = 2H$
$K_e = 1.03$
$J = 0.087\ kg \cdot m$
$D = 0.02\ Nms$
$\tau_L = 15\ Nm$

@ $t = 0$, V_f drops to 192V

Find: $\omega(t)$, $i_f(t)$, $i_a(t)$

SOL: Step 1 determine initial conditions $i_f(0)$, $i_a(0)$ & $\omega(0)$

$$i_f = \frac{V_f}{R_f} = \frac{240}{320} = 0.75\ A$$

$$V_a = R_a i_a + K_e i_f \omega$$
$$K_e i_f i_a - \tau_L - D\omega = 0$$

$$\therefore \quad i_a = \frac{V_a}{R_a} - \frac{K_e i_f \omega}{R_a}$$

$$\frac{K_e i_f V_a}{R_a} - \frac{(K_e i_f)^2}{R_a}\omega - \tau_L - D\omega = 0$$

$$\omega = \frac{K_e i_f V_a - R_a \tau_L}{(K_e i_f)^2 + R_a D}$$

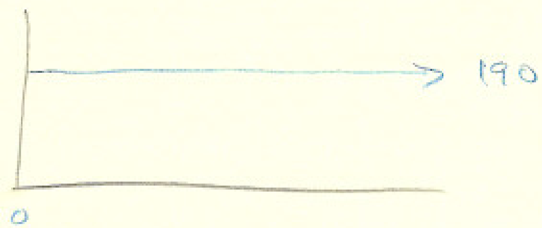
$\omega = 300.8 \text{ rad/s}$

$i_a = 27.2 \text{ A.}$

$$I_f(s) = \frac{V_f(s) - L_f\, i_f(0)}{L_f s + R_f}$$

$$V_a(s) = \frac{240}{s}$$

$$\tau_L(s) = \frac{15}{s}$$



$$I_f(s) = \frac{\frac{192}{s} + 2\times 0.75}{2s + 320}$$

$$= \frac{0.75s + 96}{s(s+160)}$$

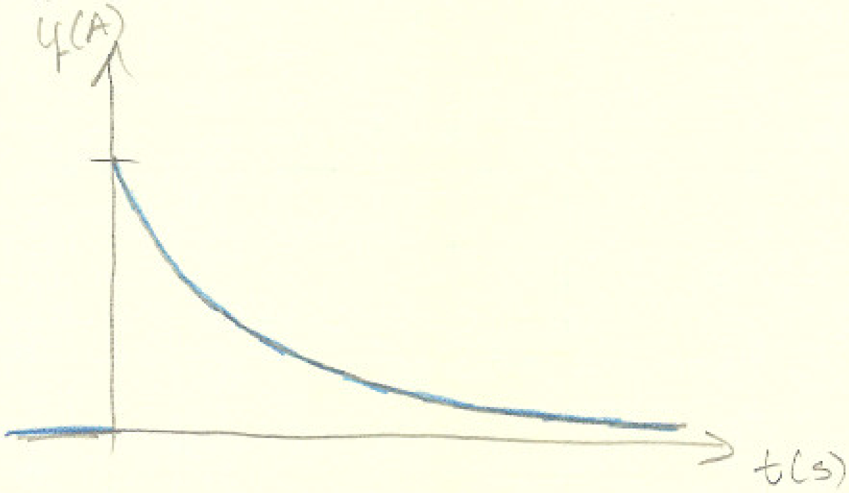
$$= \frac{0.6}{s} + \frac{0.15}{s+160}$$

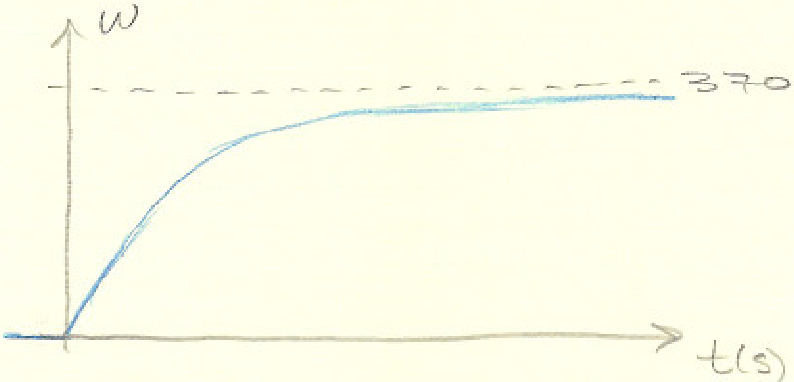
$$i_f(t) = 0.6 + 0.15e^{-160t}$$

$$\omega(s) = \frac{K_e I_f\left(V_a(s) + L_a i_a(0)\right) + \left(L_a s + R_a\right)\left(J\omega(0) - \tau\right.}{(Js + D)(L_a s + R_a) + (K_e I_f)^2}$$

$$= \frac{371.92}{s} - \frac{95.27}{(s+19.79)} + \frac{24.15}{(s+80.08)}$$

$$\omega(t) = 371.92 - 95.27e^{-19.79t} + 24.15e^{-80.08t}$$

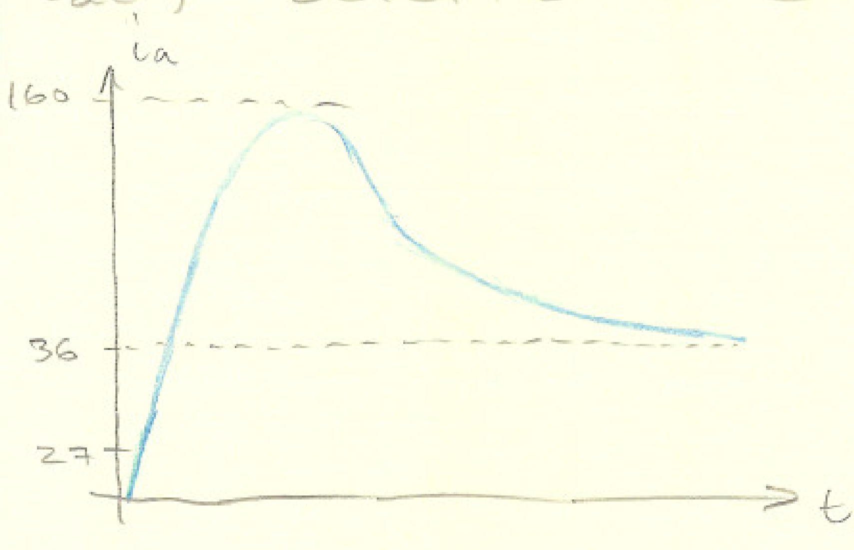




$$I_a(s) = \frac{(V_a(s) + L_a i_a(0))(Js + D) - K_e i_f (J\omega(0) - \tau_L(s)}{}$$

$$i_a(t) = \frac{36.31}{s} + \frac{262.37}{s + 19.79} - \frac{271.48}{s + 80.08}$$

$$i_a(t) = 36.31 + 262.37e^{-19.79t} - 271.48e^{-80.08t}$$



$$\frac{di_a}{dt} = 0$$

$$262.37(-19.79)e^{-19.79t} - 271.48(-8028)e^{-8028t} = 0$$

$t = 0.023$ ms

$i_a(0.023) = 160$ A.

Input $V_f(t) = R_f i_f(t) + L_f \frac{di_f(t)}{dt}$

Input $V_a(t) = R_a i_a(t) + L_a \frac{di_a(t)}{dt} + e(t)$

Disturbance

$$\tau_d(t) - \tau_L(t) - D\omega(t) = J\frac{d\omega(t)}{dt}.$$

$$e_a(t) = K_e i_f(t)\,\omega(t)$$
$$\tau_d(t) = K_a i_f(t)\, i_a(t)$$

Hall effect Current Sensor or Resistor

$$\omega(s) = \frac{K_e I_f(V_a(s) + L_a i_a(0)) + (L_a s + R_a)(J\omega(0) - \tau_L(s))}{(Js + D)(L_a s + R_a) + (K_e I_f)^2}$$

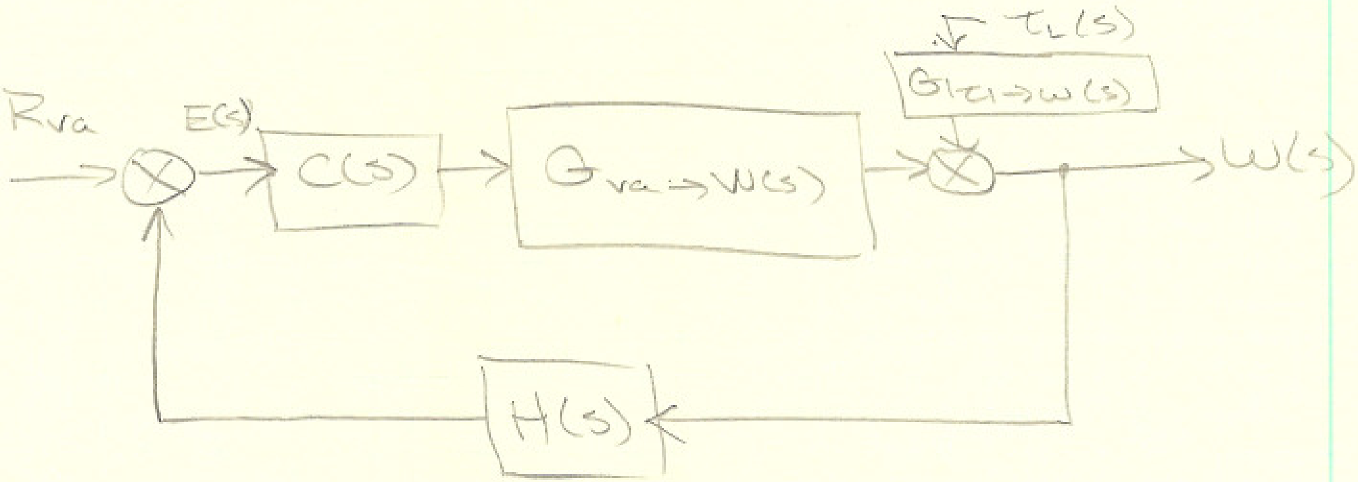
$i_f(t) = I_f$
$i_a(0) = 0$
$\omega(0) = 0$

$$= \frac{K_e I_f}{(Js + D)(L_a s + R_a) + (K_e I_f)^2} V_a(s) - \frac{(L_a s + R_a)}{(Js + D)(L_a s + R_a) + (K_e I_f)^2} \tau_L(s)$$

$$\omega(s) = G_{V_a \rightarrow \omega(s)} V_a(s)$$
$$+ G_{\tau_L \rightarrow \omega(s)} \tau_L(s)$$

<u>Poles</u> are at the roots of the char equ.

$$(Js+D)(L_a s + R_a) + (K_e I_f)^2 = 0$$



C(s) is PID.