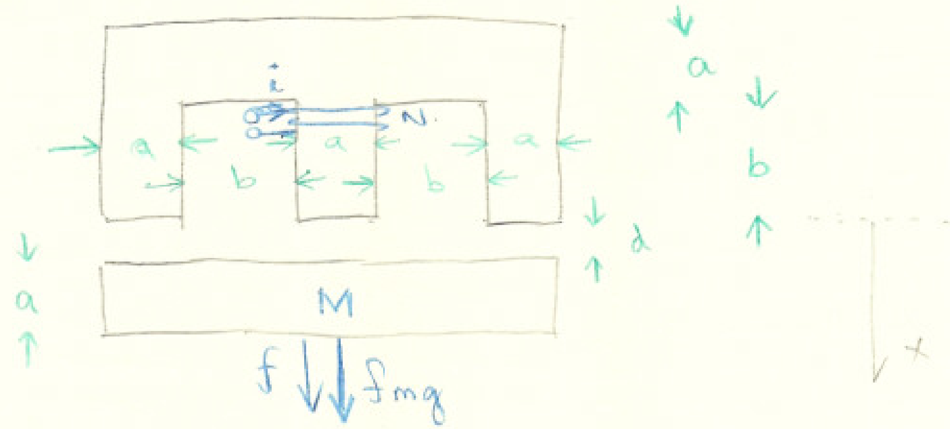
EX:



$d = 1\,cm$ $\quad$ $a = 1\,cm$

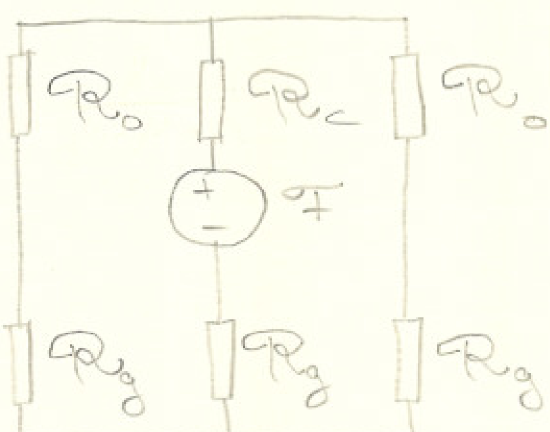
$A = 1\,cm^2$ $\quad$ $b = 5\,cm$

$\mu_r = 2000$ $\quad$ $N = 100$

$\rho = 7.85\,g/cm^3$

Find i so that M in the machine is maintained at $d = 1\,cm$

Assume that the length of the air gap is x. Then.



$$l_o = \frac{1}{2}a + b + \frac{1}{2}a + b + \frac{1}{2}a + \frac{1}{2}a + b + \frac{1}{2}a$$

$$= 3(a+b) = 18cm = 0.18m$$

$$l_c = \frac{1}{2}a + b + \frac{1}{2}a = a + b = 6cm = 0.06\,m$$

Assume that the length of the air gap is x.

$$\mathcal{R}_o = \frac{l_o}{\mu_r \mu_o A} = 7.162 \times 10^5 \text{ A}\cdot\text{t/wb}$$

$$\mathcal{R}_c = \frac{l_c}{\mu_r \mu_o A} = 2.387 \times 10^5 \text{ A}\cdot\text{t/wb}$$

$$\mathcal{R}_g = \frac{l_a}{\mu_o A} = 7.958 \times 10^9 (x)$$

$$\mathcal{F} = Ni = 100i$$

$$\mathcal{R} = \mathcal{R}_c + 0.5(\mathcal{R}_c + \mathcal{R}_g) + \mathcal{R}_g$$
$$= 1.194 \times 10^{10}(x) + 5.968 \times 10^5$$

$$L(x) = \frac{N^2}{\mathcal{R}} = \frac{100^2}{1.194(10^{10})(x) + 5.968(10^5)}$$

means the force is up

$$W_\phi(i,x) = \frac{1}{2}L(x)i^2$$

$$f = \frac{dW_\phi(i,x)}{dx} = \frac{1}{2}i^2 L'(x) = \frac{1}{2}i^2 \frac{-1.194(10^6)}{(1.194(10^6)x + 59.68)^2}$$

$$l_M = 3a + 2b = 13\text{cm}$$
$$V_M = l \cdot A = 13 \times 1 = 13\text{cm}^3$$
$$mg = \rho V g = 13 \times 7.85 \times 10^{-3} \times 9.8 = 1.000\text{ N}$$

$$|f|_{x=d} = mg$$

$$\frac{1}{2}i^2 \frac{1.194(10^6)}{(1.194(10^6)(10^{-2}) + 59.68)^2} = 1.000$$

$$\therefore\ i = 15.529 \text{ A}$$

SUMMARY

$\mathcal{F} = Ni$

$B = \mu H$

$\phi = BA$

$\mathcal{F} = \phi \mathcal{R} \qquad \mathcal{R} = \dfrac{l}{\mu A}$

$\lambda = N\phi$

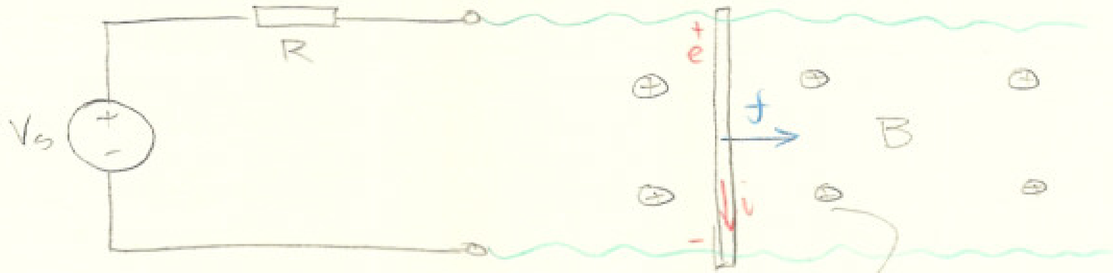
$e = \dfrac{d\lambda}{dt}$

$L = \dfrac{\lambda}{i} = \dfrac{N^2}{\mathcal{R}}$

$$\left.\begin{aligned} \lambda_1 &= \lambda_{11} + \lambda_{12} = L_{11} i_1 + L_{12} i_2 \\ \lambda_2 &= \lambda_{21} + \lambda_{22} = L_{21} i_1 + L_{22} i_2 \end{aligned}\right\} L_{12} = L_{21}$$

	Energy	Force Tourqe	Co-energy	Force tourqe
one coil	$W_\phi(\lambda, x) = \frac{1}{2}\frac{\lambda^2}{L(x)}$ $W_\phi(\lambda,\theta) = \frac{1}{2}\frac{\lambda^2}{L(\theta)}$	$f = -\dfrac{\partial W_\phi(\lambda,x)}{\partial x}$ $\tau = -\dfrac{\partial W(\lambda,\theta)}{\partial\theta}$	$W_\phi(i,x) = \frac{1}{2}L(x)\,i^2$ $W_\phi(i,\theta) = \frac{1}{2}L(\theta)\,i^2$	$f = \dfrac{\partial W_\phi(i,x)}{\partial x}$ $\tau = \dfrac{\partial W_\phi(i,\theta)}{\partial\theta}$
two coils	$W_\phi(\lambda, x) = \frac{1}{2}\Gamma_{11}\lambda_1^2 + \Gamma_{12}\lambda_1\lambda_2 + \frac{1}{2}\Gamma_{22}\lambda_2^2$ $W_\phi(\lambda,\theta) = \frac{1}{2}\Gamma_{11}\lambda_1^2 + \Gamma_{12}\lambda_1\lambda_2 + \frac{1}{2}\Gamma_{22}\lambda_2^2$	$f = -\dfrac{\partial W_\phi(\lambda_1,\lambda_2,x)}{\partial x}$ $\tau = -\dfrac{\partial W_\phi(\lambda_1\lambda_2\theta)}{\partial\theta}$	$W_\phi(i_1,i_2,x) = \frac{1}{2}L_{11}i_1^2 + L_{12}i_1i_2 + \frac{1}{2}L_{22}i_2^2$ $W_\phi(i_1,i_2,\theta) = \frac{1}{2}L_{11}i_1^2 + L_{21}i_1i_2 + \frac{1}{2}L_{22}i_2^2$	$f = \dfrac{\partial W_\phi(i_1,i_2,x)}{\partial x}$ $\tau = \dfrac{\partial W_\phi(i,i_2,\theta)}{\partial\theta}$

Dynamics of Electromechanical system (CH 3)



$$f = i\vec{l} \times \vec{B}$$

$$e = l\vec{v} \times \vec{B}$$

$$V_s = iR + e$$

$$f = m\frac{dv}{dt}$$

$$e = lvB$$

$$f = ilB$$

$$V_s = iR + lvB$$

$$i = \frac{V_s - lvB}{R}$$

$$\frac{V_s - lvB}{R} lB = m\frac{dv}{dt}$$

$$m\frac{dv}{dt} + \frac{l^2 B^2}{R} v = \frac{lBV_s}{R}$$

Dynamic model

v: state variable

We use Laplace transform ...

$$L(f(t)) = F(s)$$

$$L[f'(t)] = sF(s) - f(0)$$

$$msV(s) - mV(0) + \frac{l^2B^2}{R}V(s) = \frac{lBV_s}{R}\frac{1}{s}$$

note: $V(0) = 0$

$x(0) = \emptyset$

$$V(s) = \frac{\frac{lBV_s}{R}\frac{1}{s}}{ms + \frac{l^2B^2}{R}} = \frac{\frac{lBV_s}{mR}}{s\left(s + \frac{l^2B^2}{mR}\right)}$$

Recall: Partial fractions.

$$G(s) = \frac{N(s)}{(s+a_1)(s+a_2)(s+a_3)}$$

$$= \frac{A_1}{s+a_1} + \frac{A_2}{s+a_2} + \frac{A_3}{s+a_3}$$

$a_1 \neq a_2 \neq a_3$

$$A_1 = (s+a_1)G(s)\Big|_{s=-a_1}$$

$$V(s) = \frac{A_1}{s} + \frac{A_2}{s + \frac{l^2B^2}{mR}}$$

$$A_1 = s\left(\frac{\frac{lBV_s}{mR}}{s\left(s + \frac{l^2B^2}{mR}\right)}\right)\Bigg|_{s=0} = \frac{V_s}{lB}$$

$$A_2 = \left(s + \frac{\ell_2 B^2}{mR}\right)\left(\frac{\frac{\ell B V_s}{mR}}{s\left(s + \frac{\ell^2 B^2}{mR}\right)}\right)$$

$$V(s) = \frac{V_s}{\ell B}\frac{1}{s} - \frac{V_s}{\ell B}\frac{1}{s + \frac{\ell^2 B^2}{mR}}$$

$$\mathcal{L}^{-1} = V(t) = \frac{V_s}{\ell B} - \frac{V_s}{\ell B} e^{-\frac{\ell^2 B^2}{mR} t}$$

$$i(t) = \frac{1}{R} V_s - \frac{\ell B}{R} V(t)$$

$$= \frac{V_s}{R} e^{-\frac{\ell^2 B^2}{mR} t}$$