EX:

$V_a = V_f = 240V$
$P = 12$ hp
$R_a = 0.28\,\Omega$
$L_a = 2.81$ mH
$R_f = 320\,\Omega$
$L_f = 2H$
$K_e = 1.03$
$J = 0.087$ kg·m
$D = 0.02$ N·m·s

V_a is suddenly applied at $t=0$ with no load
Find $i_a(t)$ & $\omega(t)$.

SOL

$i_a(0) = 0$
$\omega(0) = 0$

$$\boxed{\omega(s) = \frac{K_e I_f\,(V_a(s) + L_a i_a(0)) + (L_a s + R_a)(J\omega(0) - T_L(s))}{(Js + D)(L_a s + R_a) + (K_e I_f)^2}}$$

$$= \frac{(1.03)(0.75)\left(\frac{240}{s} + 0\right) + (0.00281s + 0.28)(0 - 0)}{(0.087s + 0.02)(0.00281s + 0.28) + ((1.03)(0.75))^2}$$

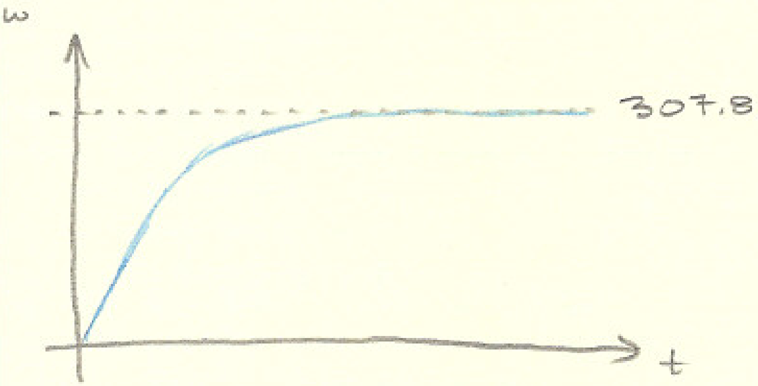
$$= \frac{7.5838 \times 10^5}{s(s + 44.482)(s + 55.391)}$$

Partial fraction expansion results in

$$= \frac{307.80}{s} - \frac{1562.9}{s + 44.482} + \frac{1255.1}{s + 55.391}$$

$\mathcal{L}^{-1}$

$$\omega(t) = 307.80 - 1562.9e^{-44.82t} + 1255.1e^{-55.391t}$$

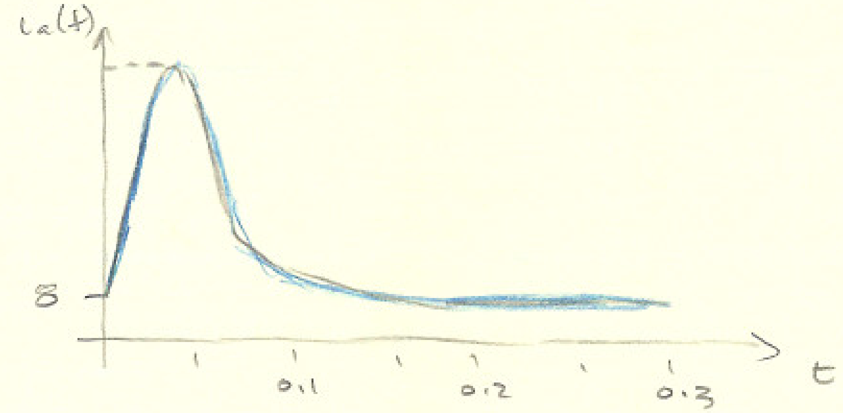


$$\boxed{I_a(s) = \frac{(V_a(s) + L_a i_a(0))(Js+D) - K_e I_f (J\omega(0) - \tau_L(s))}{(Js+D)(L_a s + R_a) + (K_e I_f)^2}}$$

$$= \frac{\left(\frac{240}{s} + 0\right)(0.087s + 0.02) + 0}{(0.087s + 0.02)(0.00281s + 0.28) + (103 \times 0.75)^2}$$

$$= \frac{7.9687}{s} + \frac{7788.8}{s + 44.482} + \frac{-7796.7}{s + 55.391}$$

$$i_a(t) = 7.9687 + 7788.8e^{-44.482t} - 7796.7e^{-55t}.$$



This current is way too high, V_a can not be applied all at once... we must slowly increase the Voltage on the armature.

Time constants

$$\tau_m = \frac{J}{D} \quad \text{(mechanical time constant)}$$

$$\tau_e = \frac{L_a}{R_a} \quad \text{(electrical time constant)}$$

$$\tau_m \gg \tau_e$$

EX

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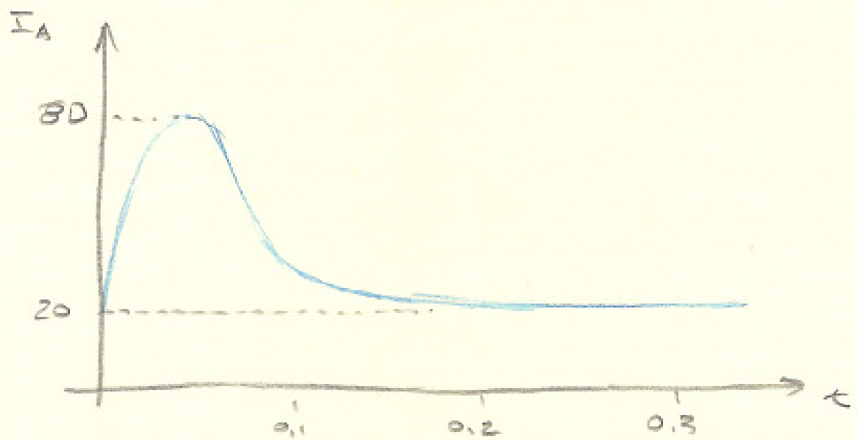
$R_f = 320\,\Omega$
$L_f = 2H$
$K_e = 1.03$
$J = 0.087$
$I_a = 37.3\,A$
$D = 0.02\,N \cdot m \cdot s$

30 V is suddenly applied at $t=0$ to load of 15 N·m
Find $i_a(t)$ & $\omega(t)$

$i_a(0) = 0$
$\omega(0) = 0$

$$I_a = 20.234 + 875.9\,e^{-44.482t} - 896.14\,e^{-55.391t}$$



Before $t=0$, 30 V has been applied to the armature winding for a long time so that it can reach steady state

At $t=0$ 60 V is applied to V_a suddenly calculate $i_a(t)$, $\omega(t)$

$$i_a(0) = ?$$
$$\omega(0) = ?$$

$$V_f(\infty) = R_f I_f(\infty)$$

$$I_f(\infty) = \frac{V_f(\infty)}{R_f} = \frac{240}{320} = 0.75$$

$$V_a(\infty) = R_a i_a(\infty) + K_e I_f \omega(\infty)$$

$$K_e I_f i_a(\infty) - \tau_L(\infty) - D\omega(\infty) = 0$$

$$\left.\begin{array}{l} i_a(0) = i_a(\infty) \\ i\omega(0) = \omega(\infty) \end{array}\right\}$$ b/c we need to see what they were at 30 V on V_a.

$$i_a(\infty) = 20.2\text{ A}$$
$$\omega(\infty) = 31.7\text{ A}$$

Then we can do the rest of the calculations.

Feild Control DC motors

$$\frac{d i_f(t)}{dt} = -\frac{R_f}{L_f} i_f(t) + \frac{1}{L_f} V_f(t)$$

$$\frac{d i_a(t)}{dt} = -\frac{R_a}{L_a} i_a(t) - \frac{K_e}{L_a} i_f(t)\omega(t) + \frac{V_a(t)}{L_a}$$

$$\frac{d\omega(t)}{dt} = \frac{K_e}{J} i_a(t) i_f(t) - \frac{1}{J}\tau_L(t) - \frac{D}{J}\omega(t)$$

With this system we assume

$$i_f(t) = I_f \qquad \text{(constant)}$$