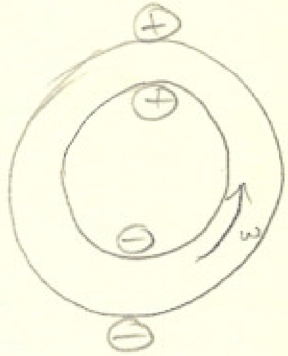
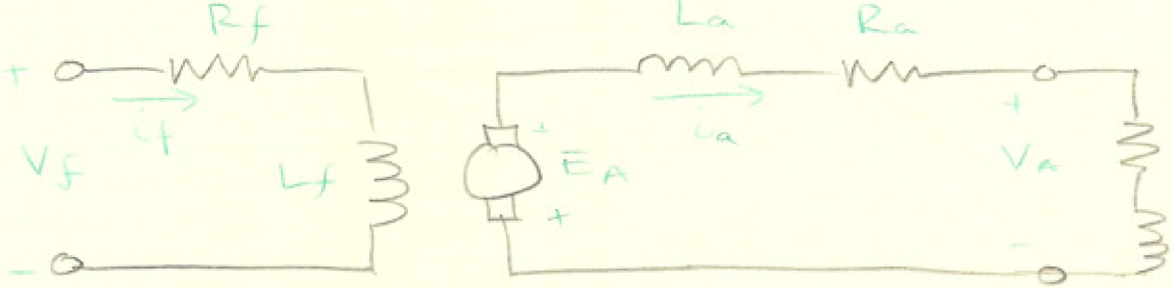




$$E_A = k\phi\omega$$
$$= K_e i_f \omega$$

Assume that ω is constant maintained by controler

$$V_f = R_f i_f + L_f \frac{di_f}{dt}$$

$$e_a = L_a \frac{di_a}{dt} + R_a i_a + R_L i_a + L_L \frac{di_a}{dt}$$

$$e_a = (L_a + L_L)\frac{di_a}{dt} + (R_a + R_L)\, i_a$$

$$K_e i_f \omega = (L_a + L_L)\frac{di_a}{dt} + (R_a + R_L) i_a$$

Steady State $t \to \infty$

$$\left.\frac{di_f}{dt}\right|_{t=\infty} = 0 \; ; \; \left.\frac{di_a}{dt}\right|_{t=\infty} = 0$$

$$V_f = R_f \, i_f(\infty)$$

$$K_e i_f(\infty)\,\omega = (R_a + R_L)\, i_a(\infty)$$

$$i_f(\infty) = \frac{V_f}{R_f} \, , \quad i_a(\infty) = \frac{K_e i_f(\infty)\,\omega}{R_a + R_L}$$

$$\tau_a = \frac{L_a + L_L}{R_a + R_L}$$

time constants..

$$\tau_f = \frac{L_f}{R_f}$$

Transient Response.

$$V_f(s) = R_f I_f(s) + L_f\left[s I_f(s) - i_f(0)\right]$$

$$K_e \omega I_f(s) = (L_a + L_L)\left[s I_a(s) - i_a(0)\right] + (R_a + R_L) I_a(s)$$

$$I_f(s) = \frac{V_f(s) + L_f \, i_f(0)}{s L_f + R_f}$$

$$\left[\frac{K_e \omega V_f(s) - K_e \omega L_f i_f(0)}{sL_f + R_f}\right] = \left[(L_a + L_L)s\right.$$

$$\left. + (R_a + R_L)\right] I_a(s) - (L_a + L_L) i_a(0)$$

$\therefore$

$$I_a(s) = \frac{K_e \omega V_f(s) + K_e \omega L_f i_f(0) - (L_a + L_L) i_a(0)(sL_f + R_f)}{[sL_f + R_f]\left[(L_a + L_L)s + (R_a + R_L)\right]}$$

EX:

$n = 1500\,rpm$
$R_a = 0.2\,\Omega$
$L_a = 2.5\,mH$
$R_f = 3\,\Omega$
$L_f = 25\,mH$
$K_e = 0.191$
$V_f = 120\,V$
$R_L = 40\,\Omega$
$L_L = 40\,mH$

Find: $e_a(t)$, $i_a(\infty)$, $i_a(t)$, $i_f(\infty)$, τ, $e_a(\infty)$

SOL:

$$i_f(\infty) = \frac{V_f}{R_f} = \frac{120V}{3\Omega} = 40\,A.$$

$$e_a(\infty) = i_a(\infty)(R_a + R_L) = K_e i_f(\infty)\,\omega$$
$$= (0.191)(40\,A)\frac{2\pi n}{60}$$

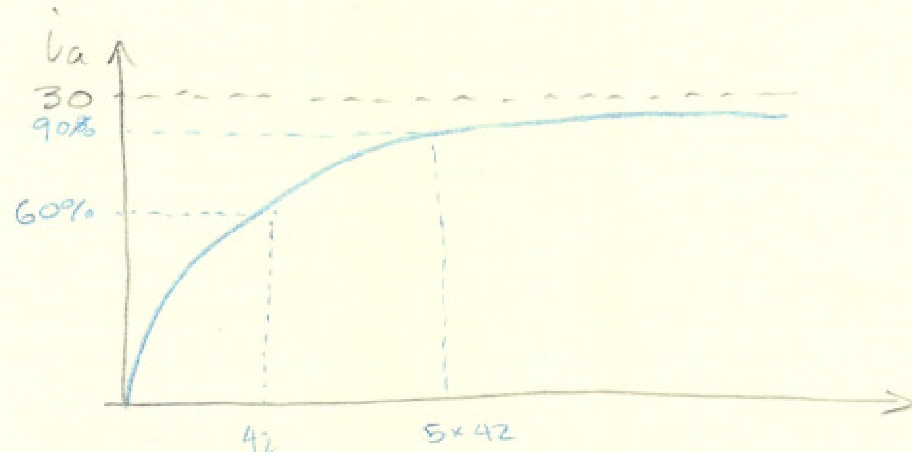
$$I_a(s) = \frac{(0.191)(157)(120) \big/ (0.025)(0.0425)}{s\left[s + \frac{3}{0.025}\right]\left[s + \frac{40.02}{0.0425}\right]}$$

$$I_a(s) = \frac{A}{s} + \frac{B}{s+120} + \frac{C}{s+945.9}$$

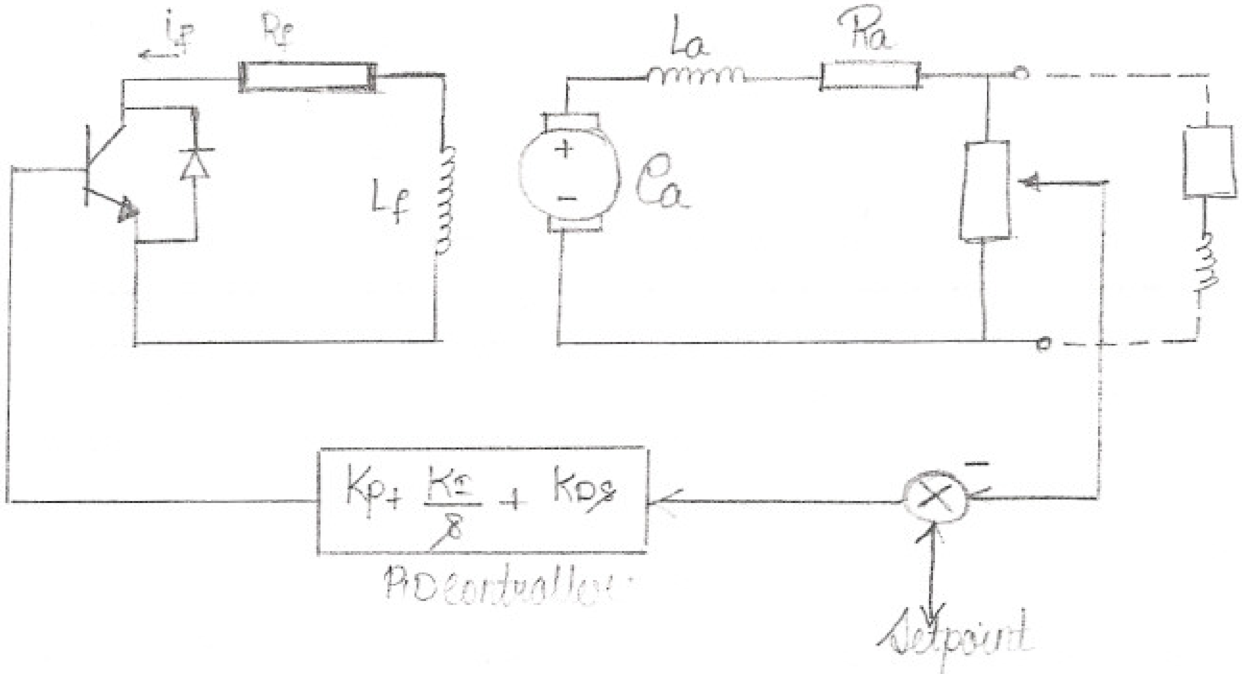
$$I_a(s) = \frac{29.8}{s} - \frac{34.8}{s+120} + \frac{4.3}{s+945.9}$$

$$i(t) = 29.8 - 34.8e^{-120t} + 4.3e^{-945.9t}$$

$$e(t) = K_e\omega\, i_f(t) = 1199.5 - 1199.5e^{-120t}$$



$\tau = 42$ time constant



i_f
R_f
L_f
L_a
R_a
e_a
$K_P + \frac{K_I}{s} + K_D s$
PID controller
Setpoint