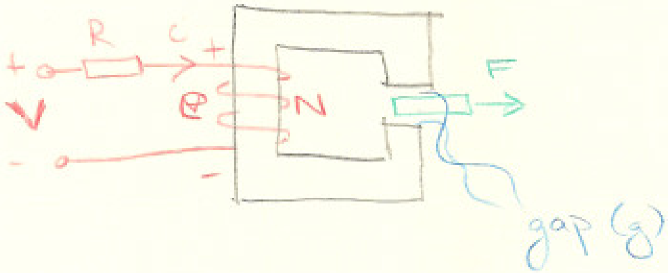
note: $A = l(d - x)$



$$P = Vi$$

$$P_{loss} = Ri^2$$

$$dW_e = P\,dt = Vi\,dt$$

$$dW_{loss} = P_{loss}\,dt = Ri^2\,dt$$

$$dW_m = f\,dx$$

$$dW_\phi = dW_e - dW_{loss} - dW_m$$

$$e = \frac{d\lambda}{dt} \Rightarrow e\,dt = d\lambda$$

$$\begin{aligned} dW_\phi &= Vi\,dt - Ri^2\,dt - f\,dx \\ &= i(V - Ri)\,dt - f\,dx \\ &= i\,e\,dt - f\,dx \end{aligned}$$

$$dW_\phi = i\,d\lambda - f\,dx$$

$$W_\phi(\lambda, x),\ i(\lambda, x),\ f(\lambda, x)$$

$$dW_\phi = \frac{\partial W_\phi(\lambda, x)}{\partial \lambda} d\lambda + \frac{\partial W_\phi(\lambda, x)}{\partial x} dx$$

$$W_\phi = \frac{1}{2} L_{11} i_1^2 + L_{12} i_1 i_2 + \frac{1}{2} L_{22} i_2^2$$

Lorentz's Law

$$\vec{f} = i\vec{l} \times \vec{B}$$

$$f = -\frac{\partial W_\phi(\lambda, x)}{\partial x}$$

$$i = \frac{\partial W_\phi(\lambda, x)}{\partial \lambda}$$

$$W_\phi(\lambda, x) = \int_0^\lambda i \, d\lambda$$

$$L = \frac{\lambda}{i} \Rightarrow \lambda = Li$$

$$W_\phi(\lambda, x) = \int_0^i L i \, di$$

$$= \frac{1}{2} L i^2 = \frac{1}{2} \frac{\lambda^2}{L}$$

$$L = \frac{\lambda}{i} = \frac{N\phi}{i} = \frac{N \frac{\mathcal{F}}{\mathcal{R}}}{i} = \frac{N \frac{Ni}{\mathcal{R}}}{i}$$

$$L = \frac{N^2}{\mathcal{R}} = \frac{N^2}{\frac{2g}{\mu_0 l(d-x)}} = \frac{N^2 \mu_0 l (d-x)}{2g}$$

$$W_\phi(\lambda, x) = \frac{1}{2} \frac{\lambda^2}{L}$$

$$= \frac{1}{2} \frac{\lambda^2}{\frac{N^2 \mu_0 l(d-x)}{2g}}$$

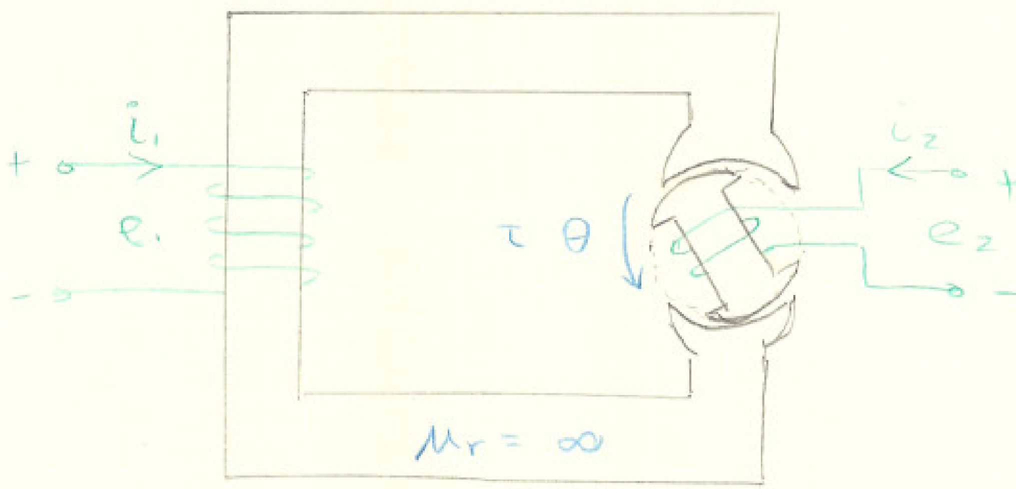
$$= \frac{g \lambda^2}{N^2 \mu_0 l (d-x)}$$

$$f = -\frac{\partial W_\phi(\lambda, x)}{\partial x}$$

$$= -\frac{g\lambda^2}{N^2\mu_0 l}\,\frac{(-1)(-1)}{(d-x)^2}$$

$$\lambda = N\phi = N\frac{\mathcal{F}}{\mathcal{R}} = \frac{N^2 i}{\mathcal{R}} = \frac{N^2 i}{\frac{2g}{\mu_0 l(d-x)}} = \frac{N^2\mu_0 l(d-x)i}{2g}$$

$$f = -\frac{\mu_0 l (Ni)^2}{4g}$$



$$dW_e = P_1\,dt + P_2\,dt$$

$$= i_1 e_1\,dt + i_2 e_2\,dt$$

$$= i_1\,d\lambda_1 + i_2\,d\lambda_2$$

$$dW_m = \tau\,d\theta$$

$$dW_\phi = dW_e - dW_m$$

$$= i_1\,d\lambda_1 + i_2\,d\lambda_2 - \tau\,d\theta$$

$$W_\phi = (\lambda_1, \lambda_2, \theta)$$

$$dW_\phi = \frac{\partial W_\phi}{\partial \lambda_1} d\lambda_1 + \frac{\partial W_\phi}{\partial \lambda_2} d\lambda_2 + \frac{\partial W_\phi}{\partial \theta} d\theta$$

$$\therefore \quad i_1 = \frac{\partial W_\phi}{\partial \lambda_1} \qquad i_2 = \frac{\partial W_\phi}{\partial i_2} \qquad \tau = -\frac{\partial W_\phi}{\partial \theta}$$

given that... $L_{11} = 0.001(3 + \cos 2\theta)$

$L_{12} = L_{21} = 0.3 \cos\theta$

$L_{22} = 30 + 10 \cos 2\theta$

Find τ for $i_1 = 0.8A$ & $i_2 = 0.01A$

note: $\tau = \dfrac{\partial \omega_\phi(i_1, i_2, \theta)}{\partial \theta}$

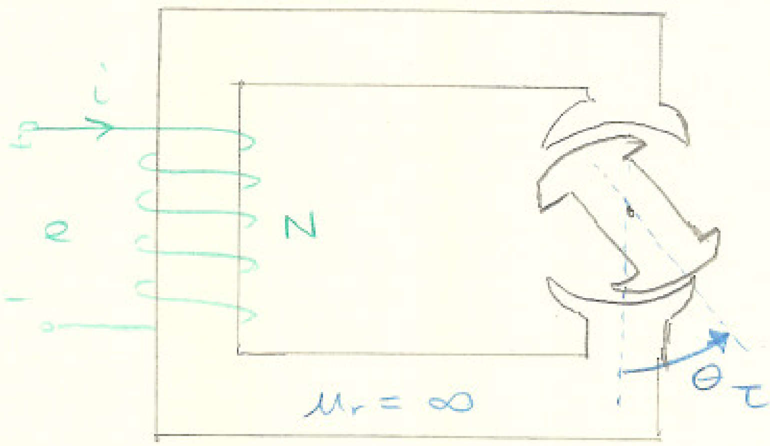
$$f = \frac{\partial \omega_\phi(i_1, i_2, x)}{\partial x}$$

$$\omega_\phi = \frac{1}{2} L_{11} i_1^2 + L_{12} i_1 i_2 + \frac{1}{2} L_{22} i_2^2$$

$$\frac{\partial \omega_\phi}{\partial \theta} = \tau = \frac{1}{2} i_1^2 \frac{dL_{11}}{d\theta} + i_1 i_2 \frac{dL_{12}}{d\theta} + \frac{1}{2} i_2^2 \frac{dL_{22}}{d\theta}$$

$$\tau = \frac{1}{2} \times 0.8^2 \times 0.001(-2\sin 2\theta) + 0.8 \cdot 0.01 \times 0.3 (-\sin\theta) + \frac{1}{2} \times 0.01^2 \times 10(-2\sin 2\theta)$$

$$= -0.00164 \sin 2\theta - 0.0024 \sin\theta$$



depth = h

$$W_\phi(\lambda, \theta) = \frac{1}{2}\frac{\lambda^2}{L}$$

$$W_\phi(i, \theta) = \frac{1}{2} L i^2$$

$$\tau = \frac{\partial W_\phi(i,\theta)}{\partial \theta}$$

$$\tau = \frac{1}{2} i^2 \frac{\partial L}{\partial \theta}$$