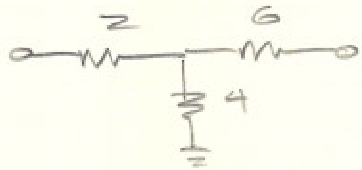
here we notice that $g_{12} = -g_{21}$

<u>H/W</u>:



<u>ans</u>:

$$\underline{G} = \begin{bmatrix} \frac{1}{6} & -\frac{2}{3} \\ \frac{2}{3} & \frac{22}{3} \end{bmatrix}$$

<u>note</u>: also prove that $\underline{G}^{-1} = \underline{H}$

INTERCONNECTION OF 2 PORT NETWORK

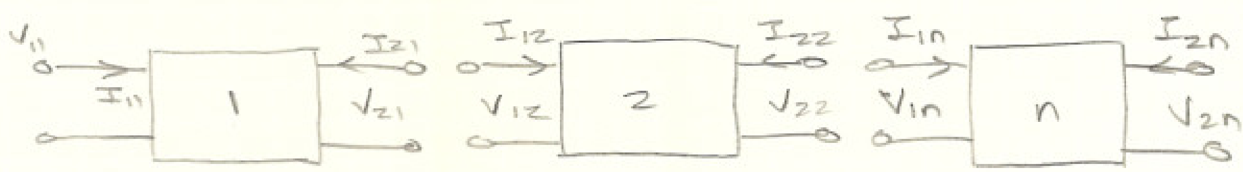
the power and versitility of a simple one port network is not obivous because of their limited behaviour as individual one ports.

from RC pairs emerged low pass, high pass and band pass/band stop filters.

Interconnections of operational amplifiers gave builting blocks which yeilded circuits of very high gain, which are extensivelly used in practical circuits.

THE CASCADE CONNECTION

consider the n 2-port networks connected in cascade. The configuration can be reduced to one that can be represented by eqivalent transmission parameter matrix "T".



Consider the 2 ports 1 and 2, and write

$$\begin{bmatrix} V_{11} \\ I_{11} \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V_{21} \\ -I_{21} \end{bmatrix}$$

$$\begin{bmatrix} V_{12} \\ I_{12} \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_{22} \\ -I_{22} \end{bmatrix}$$

At the junction of 2 ports 1 and 2, voltages must match and KCL

$$I_{12} = -I_{21},$$

then we may write

$$\begin{bmatrix} V_{21} \\ -I_{21} \end{bmatrix} = \begin{bmatrix} V_{12} \\ I_{12} \end{bmatrix}$$

henceforth

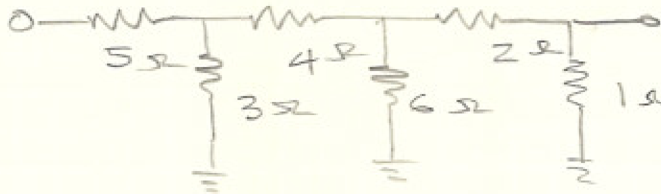
$$\begin{bmatrix} V_{11} \\ I_{11} \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_{22} \\ -I_{22} \end{bmatrix}$$

then we can find the equilent circuit.

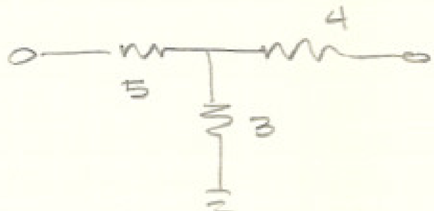
$$\begin{bmatrix} A_{eq} & B_{eq} \\ C_{eq} & D_{eq} \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$$

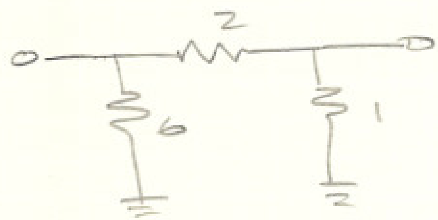
Cascade algorithm is very useful where ladder networks required reduction to a simple equivalent such as Tee or Pi networks or a simple admitance or impedance

EX: consider the following ladder network consisting of 6 resistors. Reduce it to a single Tee equivalent.



SOL (method I):





sub net 1 sub net 2

$$\underline{Z}_1 = \begin{bmatrix} (5+3) & (3) \\ (3) & (4+3) \end{bmatrix} \qquad \underline{Y}_2 = \begin{bmatrix} (\frac{1}{2}+\frac{1}{6}) & -\frac{1}{2} \\ -\frac{1}{2} & (\frac{1}{2}+\frac{1}{1}) \end{bmatrix}$$

$$\det \underline{Z} = 47 \qquad \det \underline{Y}_2 = \frac{3}{4}$$

from the tables

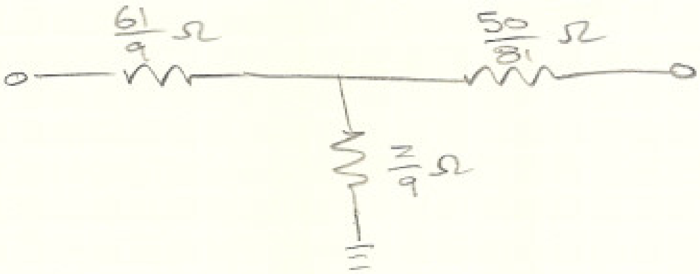
$$\underline{T}_1 = \begin{bmatrix} \frac{8}{3} & \frac{47}{3} \\ \frac{1}{3} & \frac{7}{3} \end{bmatrix} \qquad \underline{T}_2 = \begin{bmatrix} 3 & 2 \\ \frac{3}{2} & \frac{4}{3} \end{bmatrix}$$

$$\underline{T}_{eq} = \underline{T}_1 \, \underline{T}_2$$

$$\underline{T}_{eq} = \begin{bmatrix} \frac{63}{2} & \frac{236}{9} \\ \frac{9}{2} & \frac{34}{9} \end{bmatrix} \qquad \det \underline{T}_{eq} = 1$$

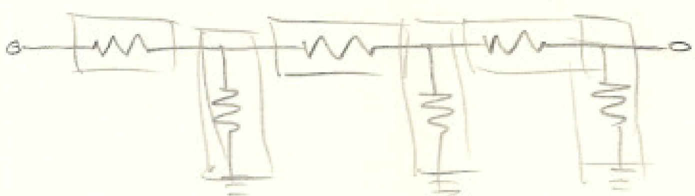
from tables, we convert $\underline{T_{eq}}$ to $\underline{Z_{eq}}$

$$\underline{Z_{eq}} = \begin{bmatrix} 7 & \frac{2}{9} \\ \frac{2}{9} & \frac{68}{81} \end{bmatrix}$$



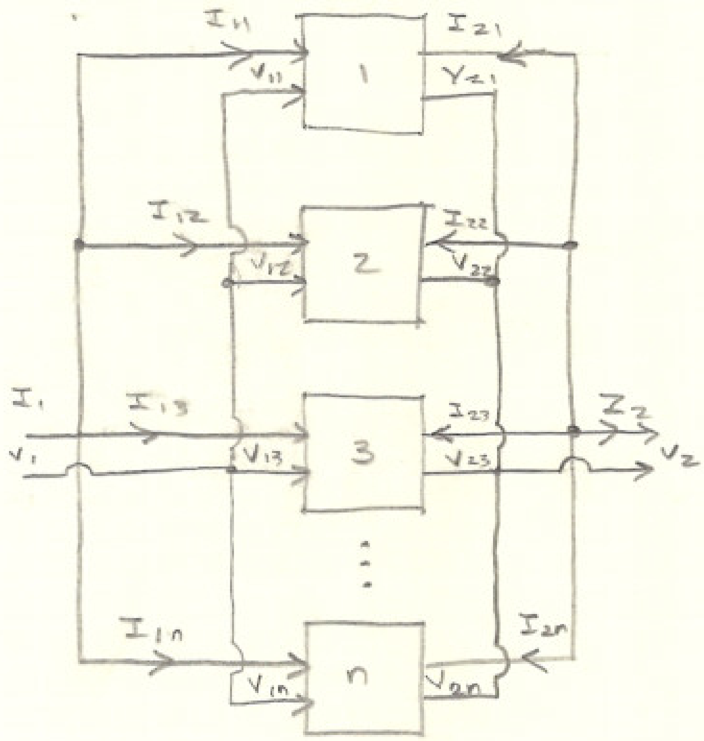
SOL (method II)

this method involves the repersentation of each resistor as either a single series or single shunt element.



$$T_{eq} = \begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \frac{1}{3} & 1 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \frac{1}{6} & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{63}{2} & \frac{236}{9} \\ \frac{9}{2} & \frac{34}{9} \end{bmatrix}$$

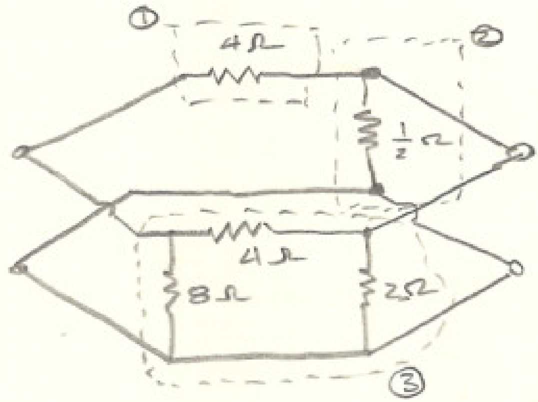


$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} V_{11} \\ V_{21} \end{bmatrix} = \begin{bmatrix} V_{12} \\ V_{22} \end{bmatrix} = \begin{bmatrix} V_{13} \\ V_{23} \end{bmatrix} = \dots = \begin{bmatrix} V_{1n} \\ V_{2n} \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \underline{Y}_1 \begin{bmatrix} V_{11} \\ V_{21} \end{bmatrix} + \underline{Y}_2 \begin{bmatrix} V_{12} \\ V_{22} \end{bmatrix} + \underline{Y}_3 \begin{bmatrix} V_{13} \\ V_{23} \end{bmatrix} + \dots + \underline{Y}_n \begin{bmatrix} V_{1n} \\ V_{2n} \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \Big[\underbrace{\underline{Y}_1 + \underline{Y}_2 + \underline{Y}_3 + \dots + \underline{Y}_n}_{\underline{Y_{eq}}} \Big] \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Ex: obtain the equivilent h-parameter matrix for the network shown below.



STEPS

1. FROM T_1 & T_2
2. use cascade algorithm to find T_{12}
3. convert $\underline{T}_{12}$ to $\underline{Y}_{12}$
4. form $\underline{Y}_3$
5. form $\underline{Y}_{eq} = \underline{Y}_{12} + \underline{Y}_3$
6. use tables to convert $\underline{Y}_{eq}$ to $\underline{h}_{eq}$

SOL:

$$\underline{T}_1 = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}, \quad \underline{T}_2 = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$\underline{T}_{12} = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 4 \\ 2 & 1 \end{bmatrix}$$

from tables

$$\underline{Y}_{12} = \begin{bmatrix} \frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{9}{4} \end{bmatrix}$$

$$\underline{Y}_3 = \begin{bmatrix} \frac{3}{8} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{3}{4} \end{bmatrix}$$

$$\underline{Y}_{eq} = \underline{Y}_{12} + \underline{Y}_3 = \begin{bmatrix} \frac{5}{8} & -\frac{1}{2} \\ -\frac{1}{2} & 3 \end{bmatrix}$$

from tables

$$\underline{h}_{eq} = \begin{bmatrix} \frac{1}{y_{11}} & -\frac{y_{12}}{y_{11}} \\ \frac{y_{21}}{y_{11}} & \frac{\Delta y}{y_{11}} \end{bmatrix}$$

$$\Delta Y = \left(\frac{5}{8}\right)(3) - \left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right) = \frac{13}{8}$$

$$h_{11} = \frac{1}{y_{11}} = \frac{8}{5}$$

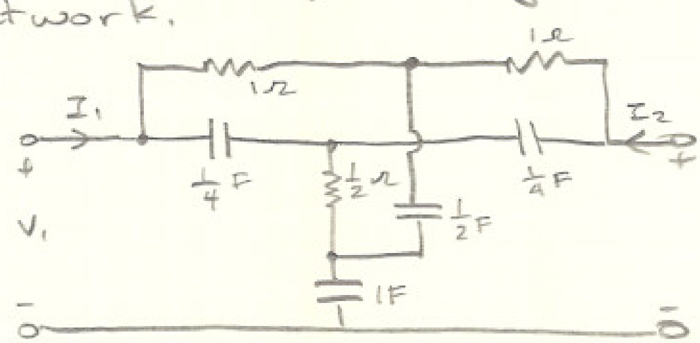
$$h_{12} = \frac{-y_{12}}{y_{11}} = \frac{4}{5}$$

$$h_{21} = \frac{y_{21}}{y_{11}} = -\frac{4}{5}$$

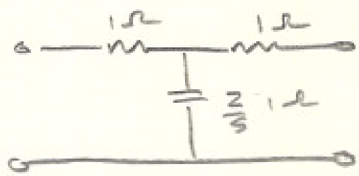
$$h_{22} = \frac{\Delta Y}{y_{11}} = \frac{13}{5}$$

$$\underline{H} = \begin{bmatrix} \frac{8}{5} & \frac{4}{5} \\ -\frac{4}{5} & \frac{13}{5} \end{bmatrix}$$

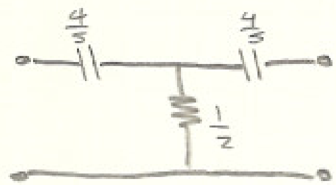
Ex: Reduce the following to a single Tee network.



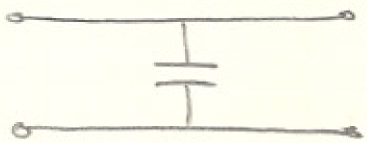
SOL:



①



②



③

① & ② are in parrellel and both are in series with the IF 2 port network, (③).

$$\underline{Z}_1 = \begin{bmatrix} 1+\frac{2}{s} & \frac{2}{s} \\ \frac{2}{s} & 1+\frac{2}{s} \end{bmatrix} = \begin{bmatrix} \frac{s+2}{s} & \frac{2}{s} \\ \frac{2}{s} & \frac{s+2}{s} \end{bmatrix}$$

from tables

$$\underline{Y}_1 = \frac{1}{s+4}\begin{bmatrix} s+2 & -2 \\ -2 & s+2 \end{bmatrix}$$

$$\underline{Z}_2 = \begin{bmatrix} \frac{4}{s}+\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{4}{s}+\frac{1}{2} \end{bmatrix}$$

from tables

$$\underline{Y}_2 = \frac{1}{8(s+4)}\begin{bmatrix} s^2+8s & -s^2 \\ -s^2 & s^2+8s \end{bmatrix}$$

① & ② are in parrellel so...

$$\underline{Y}_{12} = \underline{Y}_1 + \underline{Y}_2$$

$$\underline{Y}_{12} = \frac{1}{8(s+4)}\begin{bmatrix} s^2+16s+16 & -(s^2+16) \\ -(s^2+16) & s^2+16s+16 \end{bmatrix}$$

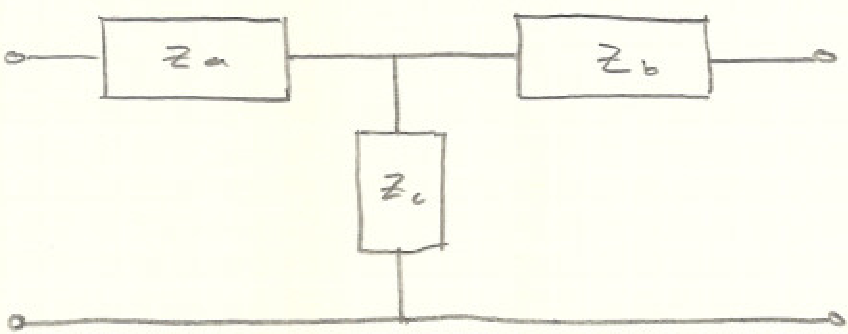
$Y_{12} \Rightarrow Z_{12}$ from tables

$$\underline{Z}_{12} = \frac{1}{4s(s+4)}\begin{bmatrix} s^2+16s+16 & s^2+16 \\ s^2+16 & s^2+16s+16 \end{bmatrix}$$

$$\underline{Z}_3 = \begin{bmatrix} \frac{1}{s} & \frac{1}{s} \\ \frac{1}{s} & \frac{1}{s} \end{bmatrix}$$

$$\underline{Z}_{13} = \underline{Z}_{12} + \underline{Z}_3 = \frac{1}{4s(s+4)}\begin{bmatrix} s^2+16s+16 & s^2+16 \\ s^2+16 & s^2+16s+16 \end{bmatrix}$$

Hence our Tee network is



$$Z_a = Z_b = \frac{4}{s+4}$$

$$Z_c = \frac{s^2+4s+32}{4s(s+4)}$$

Reciprocity

If $y_{21} = y_{12}$ and $z_{21} = z_{12}$, then the network is said to be a reciprical network. This means that both Y and Z are symetrical, for a reciprical network.

RECIPROCITY & ABCD PARAMETERS.

consider $\begin{vmatrix} A & B \\ C & D \end{vmatrix} = AD - BC$

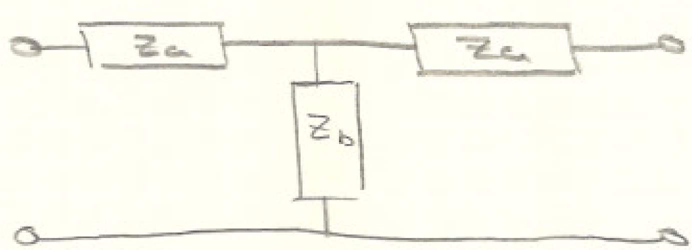
where $A = \dfrac{z_{11}}{z_{21}}$, $B = \dfrac{|Z|}{z_{21}}$, $C = \dfrac{1}{z_{21}}$, $D = \dfrac{z_{22}}{z_{21}}$

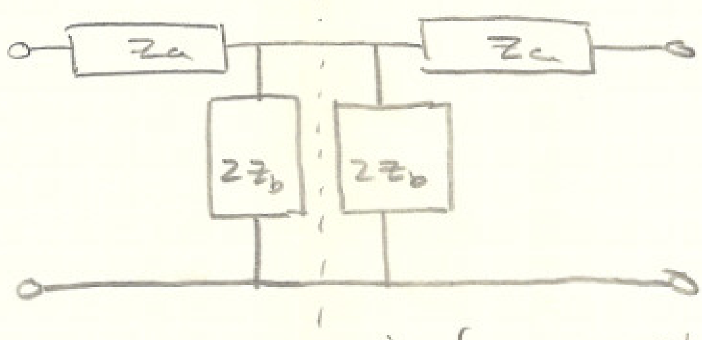
now $\begin{vmatrix} A & B \\ C & D \end{vmatrix} = \dfrac{z_{11}}{z_{21}}\dfrac{z_{22}}{z_{21}} - \left\{\dfrac{z_{11}z_{22} - z_{12}z_{21}}{z_{21}}\right\}\dfrac{1}{z_{21}} = \dfrac{z_{12}}{z_{21}}$

if $z_{12} = z_{21}$ $\quad |I| = 1$

CONCEPT OF SYMETRICAL NETWORKS.

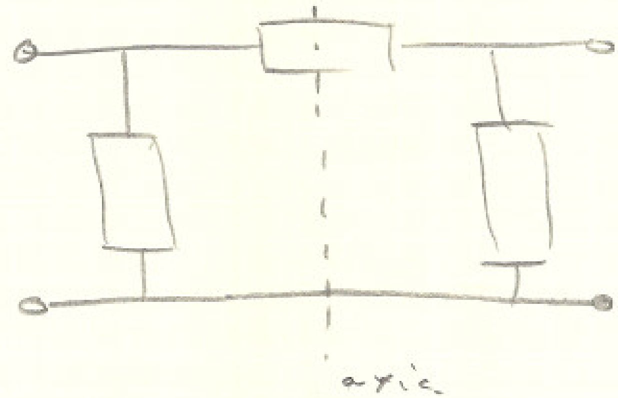
Ex:



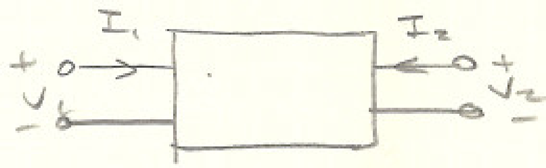


axis of symmetry

Ex:



APPLICATION OF h-PARAMETERS.



h parameters are defined as $h_i = h_{11}$, $h_r = h_{12}$, $h_f = h_{21}$ and $h_o = h_{22}$.

where, i, r, f, and o, stand for input, reverse, forward and output.

The second subscript specifies the type of connection use

e = common emitter

c = common collector

b = common base

COMMON EMITTER

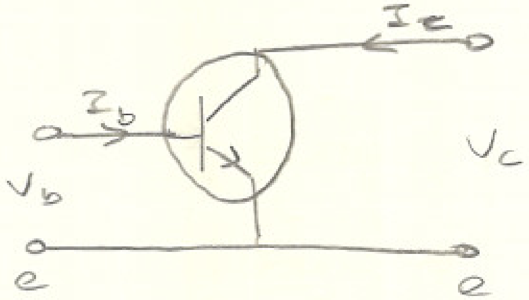
h_{ie} = base input impedance.

h_{re} = reverse voltage feedback ratio

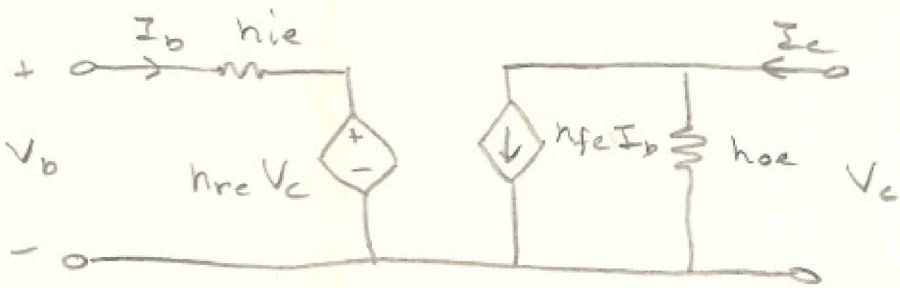
h_{fe} = Base collector current gain

h_{oe} = Output admittance.

The above parameters are calculated using open or closed circuit techniques, as was done earlier.



$$V_b = h_{ie} I_b + h_{re} V_c$$
$$V_c = h_{fe} I_b + h_{oe} V_c$$



Typical values for h parameter (CE)

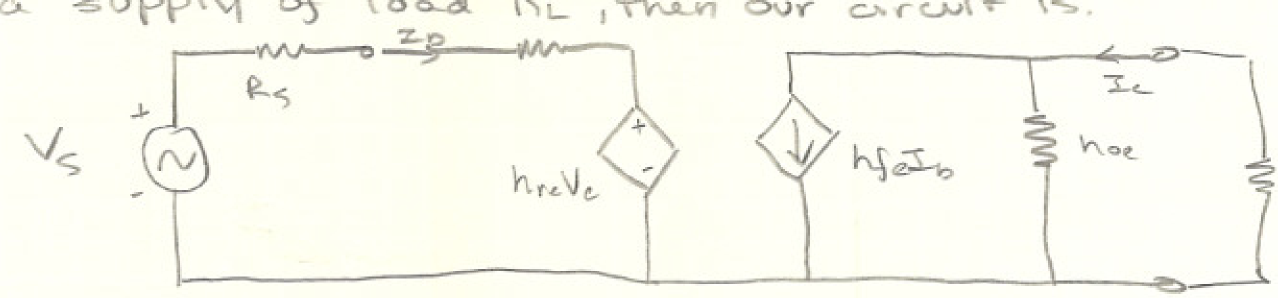
$h_{ie} = 6k\Omega$

$h_{fe} = 200$

$h_{re} = 1.5 \times 10^{-4}$

$h_{oe} = 8\mu S$

if a transistor connected to an ac source, and a supply of load R_L, then our circuit is.



$V_c = -R_L I_c$

$I_c = h_{fe} I_b + h_{oe} V_c$

$I_c = h_{fe} I_b - h_{oe} R_L I_c$

$(1 + h_{oe}) I_c = h_{fe} I_b$

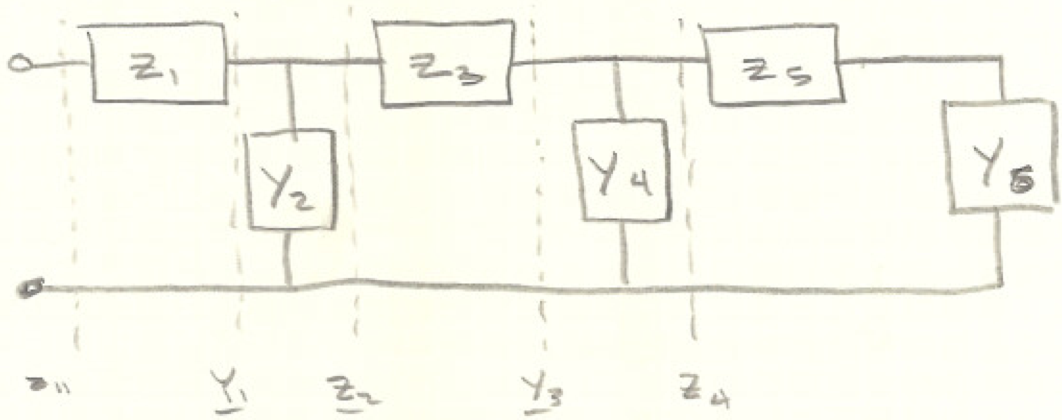
$A_i =$ current gain $= \dfrac{I_c}{I_b}$

$A_v =$ voltage gain $= \dfrac{V_c}{V_B}$

$$A_i = \frac{h_{fe}}{1 + h_{oe} R_L}$$

$$A_v = \frac{-h_{fe} R_L}{h_{ie} + (h_{ie} h_{oe} - h_{re} h_{fe}) R_L}$$

LADDER NETWORKS



Calculating the impedance z_{11} at port 1

Impedance of the last 2 arms $= Z_4 = Z_5 + \dfrac{1}{Y_6}$

admitance of the last 3 arms $= Y_3 = Y_4 + \dfrac{1}{Z_4}$

Impedance " " " 4 " $= Z_2 = Z_3 + \dfrac{1}{Y_3}$

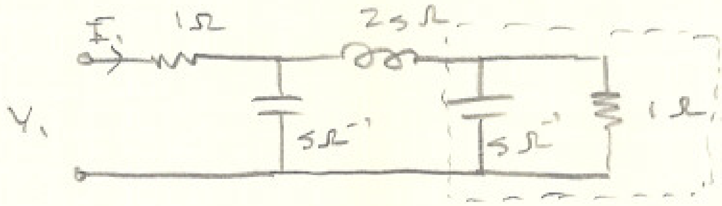
Admitance " " " 5 " $= Y_1 = Y_2 + \dfrac{1}{Z_2}$

Impedance " " " 6 " $= Z_{11} = Z_1 + \dfrac{1}{Y_1}$

$\therefore$

$$z_{11} = z_1 + \cfrac{1}{Y_2 + \cfrac{1}{z_3 + \cfrac{1}{Y_4 + \cfrac{1}{z_5 + \cfrac{1}{Y_6}}}}}$$

Ex: find z_{11} of.



$Y_4 = s+1$

$$z_{11} = \frac{2s^3 + 4s^2 + 4s + 2}{2s^2 + 2s^2 + 2s + 1}$$

Ex: Synthises $\quad z(s) = \dfrac{2s^2 + 12s + 16}{s^2 + 4s + 3}$

SOL:

$$\begin{array}{r|l} s^2+4s+3 & 2s^2+12s+16 \\ \hline 2 & \underline{2s^2+8s+6} \\ & \quad 4s+10 \end{array}$$

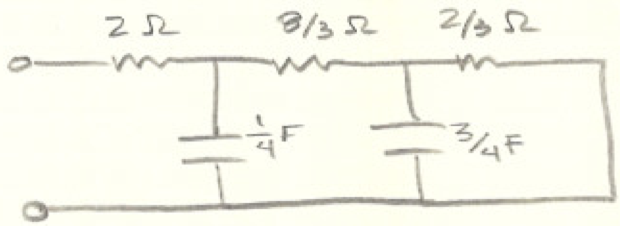
$$z(s) = 2 + \cfrac{1}{\cfrac{s^2+4s+3}{4s+10}}$$

$$\begin{array}{r|l} 4s+10 & s^2+8s+6 \\ \hline 1/4\,s & \underline{s^2 + \frac{10}{4}s} \\ & \quad \frac{3}{2}s + 3 \end{array}$$

$$Z(s) = 2 + \cfrac{1}{\frac{1}{4}s + \cfrac{1}{\cfrac{4s+10}{\frac{3}{2}s+3}}}$$

$$\begin{array}{r|l} 3/2s + 3 & 4s + 10 \\ \hline \frac{8}{3} & 4s + 8 \\ & \overline{\quad\quad 2} \end{array}$$

$$\therefore \quad Z(s) = 2 + \cfrac{1}{\frac{1}{4}s + \cfrac{1}{\frac{8}{3} + \cfrac{1}{\cfrac{3/2s+3}{2}}}}$$



note: multiply back to check.

CAUR SYNTHESIS

we need to synthesise an impedance $Z(s)$ such that

$$Z(s) = k\left\{\frac{s^m + a_{m-1}s^{m-1} + \ldots + a_0}{s^n + b_{n-1}s^{n-1} + \ldots + b_0}\right\}$$

or $\longrightarrow \boxed{\frac{N(s)}{D(s)}} \longrightarrow$ where k is positive from the example done last, last day.

expanding the $Z(s)$ is the cauer 1 network

this is easy to check, by finding the total impedence of the network.

This procedure is known as cauer procedure and the network is known as cauer network.

When we arrange the functions $N(s)$ & $D(s)$ in desending order, the resulting network is called a cauer I network

If $N(s)$ and $D(s)$ are arranged in ascending order, the resulting network is called a cauer II network.

EX:

$$Z(s) = \frac{2s+s^3}{1+s^2}$$ ascending powers.

find the cauer II network.

SOL:

$$\begin{array}{r|l} 1+s^2 & 2s+s^3 \\ \hline 2s & 2s+2s^3 \\ & \overline{\quad -s^3} \end{array}$$

negative sign indicates that $z=0$, then

$$Z(s) = \cfrac{1}{\cfrac{1+s^2}{2s+s^3}}$$

$$\begin{array}{r|l} 2s+s^3 & 1+s^2 \\ \hline \frac{1}{2s} & \underline{1+\tfrac{1}{2}s^2} \\ & \tfrac{1}{2}s^2 \end{array}$$

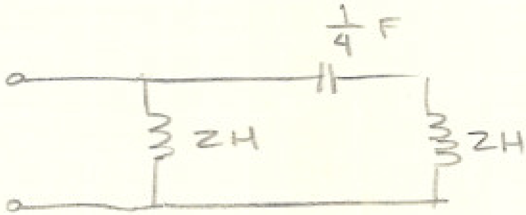
$$Z(s) = 0 + \cfrac{1}{\cfrac{1}{2s} + \cfrac{s^2}{4s+2s^3}}$$

$$Z(s) = 0 + \cfrac{1}{\cfrac{1}{2s} + \cfrac{1}{\cfrac{4s+2s^3}{s^2}}}$$

$$Z(s) = 0 + \cfrac{1}{\cfrac{1}{2s} + \cfrac{1}{\cfrac{4}{s} + 2s}}$$

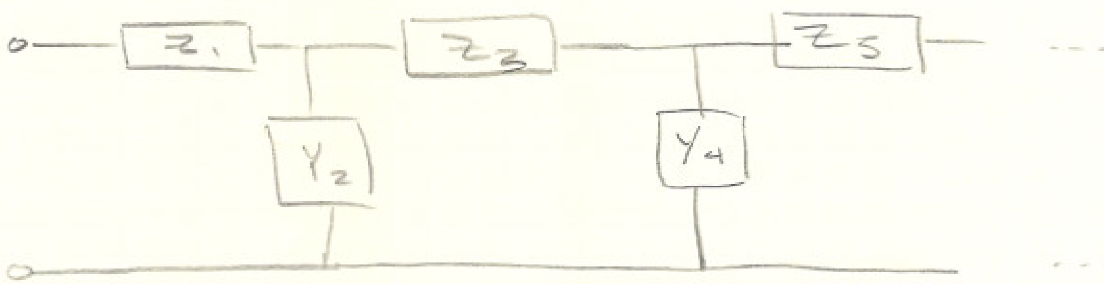
$$Z(s) = 0 + \cfrac{1}{\cfrac{1}{2s} + \cfrac{1}{\cfrac{4}{s} + \cfrac{1}{2s}}}$$

the following results in



GENERAL LC CAUER NETWORKS

there are 2 networks associated with an impedance $Z(s)$ which are realizable. Cauer networks are ladder networks which have general form shown below.

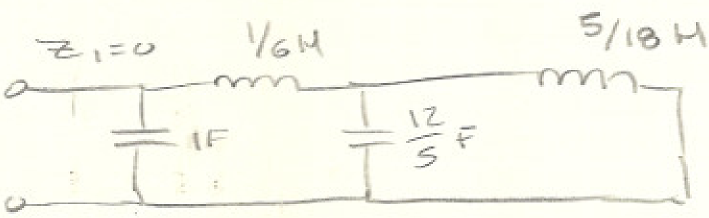


$$Z = Z_1 + \cfrac{1}{Y_2 + \cfrac{1}{Z_3 + \cfrac{1}{Y_4 \cdots \text{etc.}}}}$$

Ex:

$$Z(s) = \frac{s(s^2+4)}{(s^2+1)(s^2+9)}$$

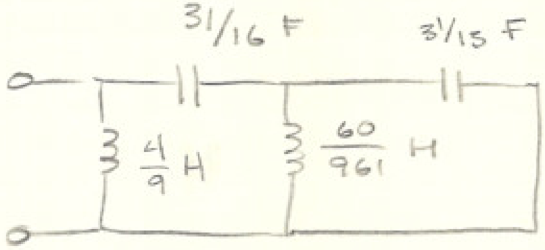
SOL:



2^{nd} cauer network is obtained from:

$$Y(s) = \frac{9 + 10s^2 + s^4}{4s + s^3}$$

$$Y(s) = \frac{9}{4s} + \cfrac{1}{\cfrac{16}{31s} + \cfrac{1}{\cfrac{961}{60s} + \cfrac{1}{\cfrac{15}{31s}}}}$$



Ex: find Cauer realizations of

$$z(s) = \frac{s^4 + 10s^2 + 9}{s^3 + 2s}$$

$$\begin{array}{r|l} s^3 + 2s & s^4 + 10s^2 + 9 \\ s & \underline{s^4 + 2s^2} \\ & \quad\quad 8s^2 + 9 \end{array}$$

$$Z(s) = s + \cfrac{1}{\cfrac{s^3 + 2s}{8s^2 + 9}}$$

$$\begin{array}{r|l} 8s^2 + 9 & s^3 + 2s \\ \frac{1}{8}s & \underline{s^3 + \frac{9}{8}s} \\ & \quad \frac{7}{8}s \end{array}$$

$$Z(s) = s + \cfrac{1}{\frac{1}{8}s + \cfrac{1}{\underline{8s^2 + 9}}}$$

$$Z(s) = s + \cfrac{1}{\frac{1}{8}s + \cfrac{1}{\frac{64}{7}s + \cfrac{1}{\frac{7s}{72}}}}$$