H/W:

FIND THE INVERSE OF:

(A) $F(s) = \frac{s}{(s+1)(s+2)^2}$

ans for (A)

$$f(t) = \left\{ -e^{-t} + e^{-2t} + 2te^{-2t} \right\}$$

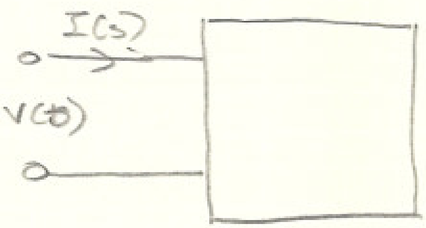
(B)

$$F(s) = \frac{16}{s^2(s+4)}$$

ans for (B)

$$f(t) = \left\{ 4t - te^{-4t} \right\} u(t)$$

TWO PORT NETWORKS

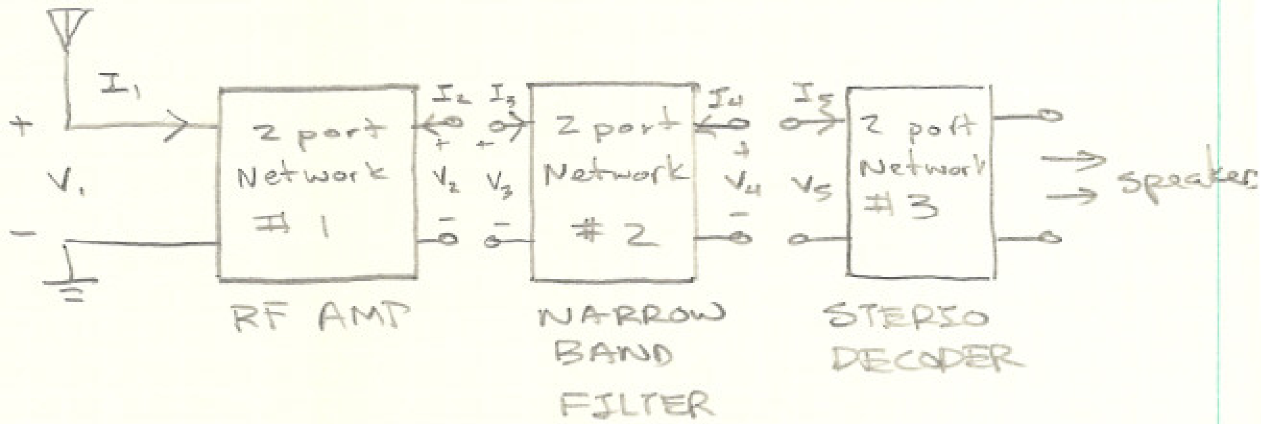


$$\frac{V(s)}{I(s)} = Z(s)$$

$$\frac{I(s)}{V(s)} = Y(s)$$

this is known as a one port network.

a practical example.



moduler Reciever design by interconnected 2 ports.

ANALYSIS OF 2 PORT NETWORKS

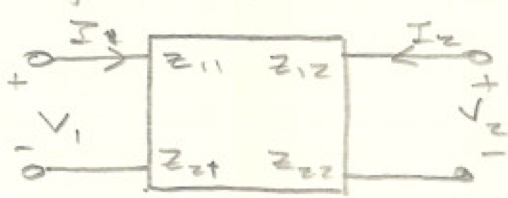
1. OPEN CIRCUIT IMPEDANCE PARAMETERS OR Z PARAMETERS.

   a set of equations that completely describe the 2 port, z-parameters network may be written as.

$$V_1 = z_{11} I_1 + z_{12} I_2$$

$$V_2 = z_{21} I_1 + z_{22} I_2$$

   it is assumed that all values are Laplace Transform quantities.



when $I_2 = 0$, then

$$\left.\frac{V_1(s)}{I_1(s)}\right|_{I_2=0} = z_{11}$$ (in port driving point impedance)

$$\left.\frac{V_2(s)}{I_1(s)}\right|_{I_2=0} = z_{21}$$ (open circuit transfer impedance)

when $I_1 = 0$

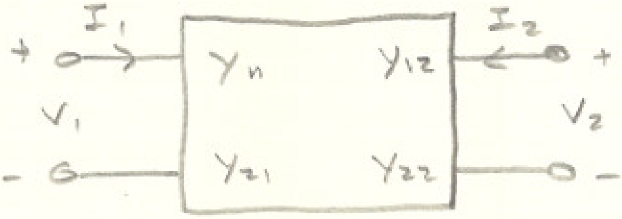
$$\left.\frac{V_1(s)}{I_1(s)}\right|_{I_1=0} = z_{12}$$ (open circuit transfer impedance)

$$\left.\frac{V_2(s)}{I_1(s)}\right|_{I_2=0} = z_{22}$$ (output driving pt impedance looking into port 2)

From Above

$$\begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} = \underbrace{\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}}_{\text{impedance matrix}} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

2. SHORT CIRCUIT ADMITANCE PARAMETERS OR Y - PARAMETERS



$$I_1 = y_{11} V_1 + y_{12} V_2$$

$$I_2 = y_{21} V_1 + y_{22} V_2$$

$$\left.\frac{I_1}{V_1}\right|_{V_2=0} = y_{11}$$ (driving pt admitance)

$$\left.\frac{I_2}{V_1}\right|_{V_2=0} = y_{21}$$ transfer admitance

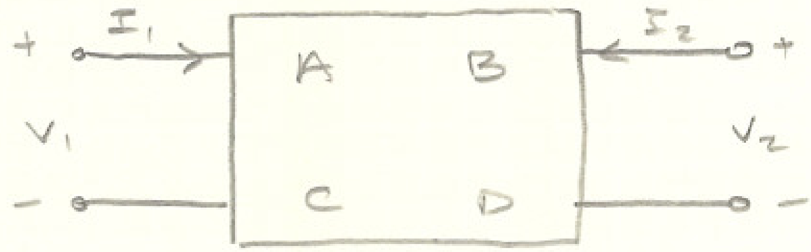
$$\left.\frac{I_1}{V_2}\right|_{V_1=0} = y_{12}$$ transfer admitance

$$\left.\frac{I_2}{V_2}\right|_{V_1=0} = y_{22}$$ driving pt admitance

and we have

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

3. CHAIN OR ABCD MATRIX (TRANSFER PARAMETERS)



equations are

$$V_1 = A V_2 - B I_2$$
$$I_1 = C V_2 - D I_2$$

in matrix form

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

EX:

$$V_1 = z_{11} I_1 + z_{12} I_2$$
$$V_2 = z_{21} I_1 + z_{22} I_2$$

$$\frac{V_1}{z_{11}} = I_1 + \frac{z_{12} I_2}{z_{11}}$$

$$\frac{V_2}{z_{21}} = I_1 + \frac{z_{22} I_2}{z_{21}}$$

$$\frac{V_1}{z_{11}} - \frac{V_2}{z_{21}} = \left(\frac{z_{12}}{z_{11}} - \frac{z_{22}}{z_{21}}\right) I_2$$

$$\frac{V_1}{z_{11}} = \frac{V_2}{z_{21}} + \left(\frac{z_{12}z_{21} - z_{11}z_{22}}{z_{11}z_{21}}\right) I_2$$

$$V_1 = \frac{z_{11}}{z_{21}} V_2 - \frac{|z|}{z_{21}} I_2$$

$$\therefore \quad A = \frac{z_{11}}{z_{21}} \qquad B = \frac{|z|}{z_{21}}$$

and now solving for C & D.

$$V_2 = z_{21} I_1 + z_{22} I_2$$
$$V_2 - z_{22} I_2 = z_{21} I_1$$

$$I_1 = \frac{1}{z_{21}} V_2 - \frac{z_{22}}{z_{21}} I_2$$

$$C = \frac{1}{z_{21}} \qquad D = \frac{z_{22}}{z_{21}}$$

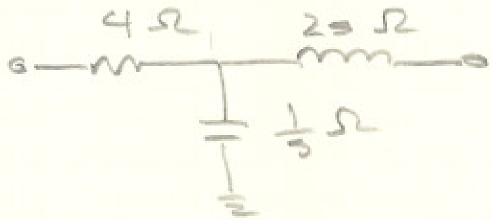
Z PARAMETERS & Y-PARAMETERS

$$\underline{Y} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = Z^{-1}$$

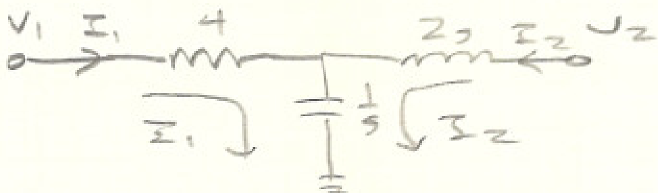
$$\underline{Y} = \frac{1}{|Z|} \begin{bmatrix} z_{22} & -z_{12} \\ -z_{21} & z_{11} \end{bmatrix}$$

note: adjoint method.

EX: determine the z-parameters for the s domain network



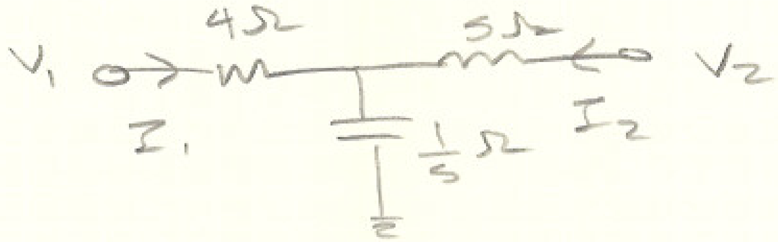
SOL:



$$V_1 = \left(4 + \frac{1}{s}\right) I_1 - \left(\frac{1}{s}\right) I_2$$

$$V_2 = \left(-\frac{1}{s}\right) I_1 + \left(2s + \frac{1}{s}\right) I_2$$

EX: find the z parameters of



SOL:

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{4s+1}{s}$$

$$z_{21} = \frac{1}{s} = z_{12}$$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{2s^2+1}{s}$$

$$\therefore \quad Z(s) = \begin{bmatrix} \frac{4s+1}{s} & \frac{1}{s} \\ \frac{1}{s} & \frac{2s^2+1}{s} \end{bmatrix}$$

EX: for the same circuit find the y parameters

$$I_1 = y_{11} V_1 + y_{12} V_2$$
$$I_2 = y_{21} V_1 + y_{22} V_2$$

SOL: $$\left.\frac{I_1}{V_1}\right|_{V_2=0} = y_{11} = \frac{2s^2+1}{8s^2+2s+4}$$

$$\left.\frac{I_1}{I_2}\right|_{V_2=0} = -D$$

$$V_x = -2sI_2$$

$$I_1 + I_2 = (V_x)s$$

divide by I_2, sub for V_x, we get

$$\frac{I_1}{I_2} + 1 = -2s^2$$

$\therefore \quad -D = -2s^2 - 1 \quad \therefore \quad D = 2s^2 + 1$

$$\frac{V_1 - V_x}{4} = I_1 \,, \quad V_x = -2sI_2$$

and

$I_1 = sV_x - I_2 \quad$ from $\quad I_1 + I_2 = sV_x$

$\therefore \quad I_1 = -(2s^2+1)I_2$

$$\frac{V_1 + 2sI_2}{4} = -(2s^2+1)I_2$$

$$V_1 + 2sI_2 = -4(2s^2+1)I_2$$

$$\left.\frac{V_1}{I_2}\right|_{V_2=0} = -4(2s^2+1)-2s = -B$$

$$\therefore \quad B = 8s^2+2s+1$$

$$\underline{T}(s) = \begin{bmatrix} 4s+1 & 8s^2+2s+1 \\ s & 2s^2+1 \end{bmatrix}$$

HYBRID PARAMETERS (h-parameters)
AND G PARAMETERS.

$$G = H^{-1}$$

the h-parameters are useful in the analysis of electronic circuits, containing transistors etc... These parameters are a mixed set.

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

in matrix form

$$\underline{H} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$$

The values of the individual h parameters can be determined from measurements or from calculations under short and open circuit conditions.

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = \_\_\_\_ \ \Omega$$

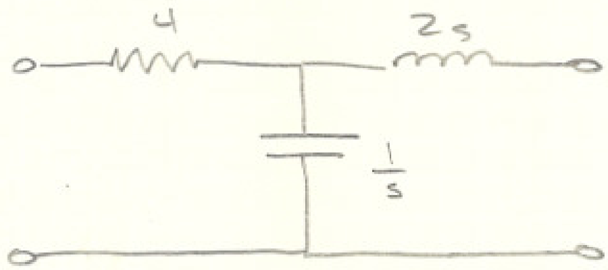
$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \_\_\_\_ \text{(voltage gain)}$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = \_\_\_\_ \text{(current gain)}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \_\_\_\_ \ S \ \mho$$

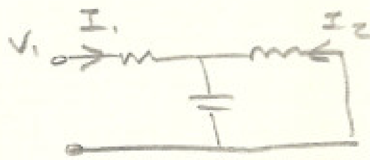
unit of seimens for admitance

EX: given



find the H-parameters.

SOL: Circuit A. $V_2 = 0$ Circuit B. $I_1 = 0$



from circuit ⓐ with $V_2(s)=0$

$$h_{11} = \frac{V_1}{I_1}\bigg|_{V_2=0} = 4 + \frac{2s\left(\frac{1}{s}\right)}{2s+\left(\frac{1}{s}\right)}$$

$$= \frac{8s^2+2s+4}{2s^2+1}$$

the value of $I_2(s)$ can be obtained from current divider

$$I_2(s) = \frac{\left(-\frac{1}{s}\right) I_1}{2s+\left(\frac{1}{s}\right)}$$

$$I_2(s) = \frac{-1}{2s^2+1} I_1(s)$$

$$\therefore \quad h_{21} = \frac{-1}{2s^2+1}$$

The other Z parameters are determined from open circuit condition with $I_1=0$ as shown in figure Ⓑ, so that

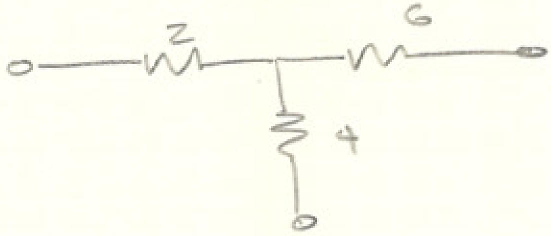
$$V_1 = \left(\frac{1/s}{2s + 1/s}\right) V_2(s)$$

$$h_{12} = \frac{V_1}{V_2}\bigg|_{I_1=0} = \frac{1}{2s^2+1}$$

$$h_{22} = \frac{I_2}{V_2}\bigg|_{I_1=0} = \frac{s}{2s^2+1}$$

$$\therefore \quad \underline{H}(s) = \begin{bmatrix} \frac{8s^2+2s+4}{2s^2+1} & \frac{1}{2s^2+1} \\ \frac{-1}{2s^2+1} & \frac{s}{2s^2+1} \end{bmatrix}$$

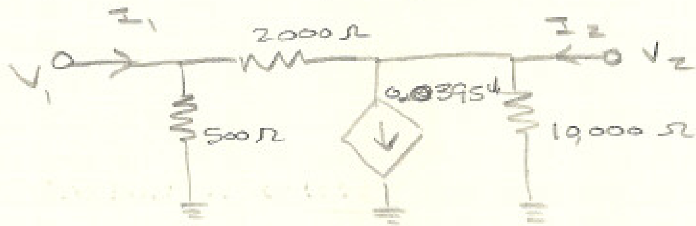
<u>H/W</u>: determine h parameters.



<u>SOL</u>:

$$\underline{H} = \begin{bmatrix} \frac{22}{5} & \frac{2}{5} \\ -\frac{2}{5} & \frac{1}{10} \end{bmatrix}$$

Ex: find the y-parameters of

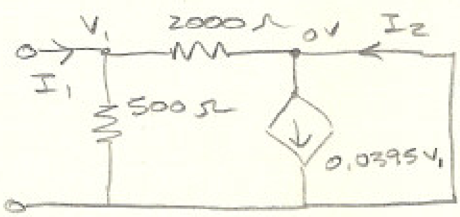


SOL

$$I_1 = y_{11} V_1 + y_{12} V_2$$

$$I_2 = y_{21} V_1 + y_{22} V_2$$

$$\left.\frac{I_1}{V_1}\right|_{V_2=0} = y_{11} \qquad \left.\frac{I_2}{V_1}\right|_{V_2=0} = y_{21}$$



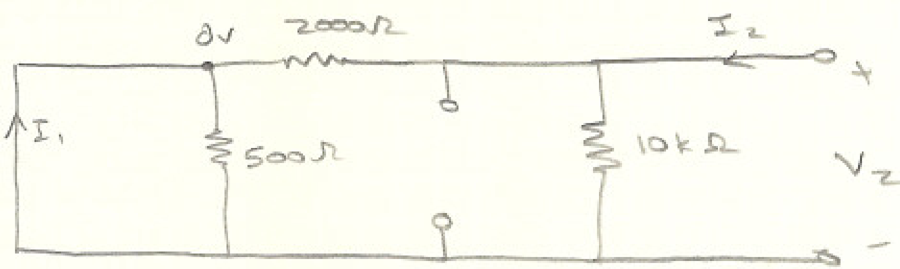
10 kΩ resister is gone b/c it is shorted out.

$$\left. I_1 \right|_{V_2=0} = \frac{V_1}{500} + \frac{V_1}{2000}$$

$$\frac{I_1}{V_1} = \frac{2000 + 500}{(2000)(500)} = y_{11} = 2.5 \text{ m℧}$$

$$I_2 = \frac{0.0395 V_1}{\frac{V_1}{2000}} = 39 \text{ m℧}$$

for y_{12} and y_{22}, when $V_1 = 0$



$$I_2 = \frac{V_2}{10{,}000} + \frac{V_2}{2000}$$

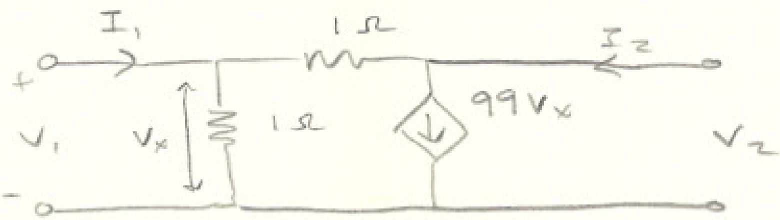
$$\left.\frac{I_2}{V_2}\right|_{V_1=0} = y_{22} = 0.6 \text{ m℧}$$

$$I_1 = -\frac{V_2}{2000}$$

$$\left.\frac{I_1}{V_2}\right|_{V_1=0} = y_{12} = -0.5 \text{ m℧}$$

$$\underline{Y} = \begin{bmatrix} 2.5 & -0.5 \\ 39 & 0.6 \end{bmatrix} \qquad \text{where } ℧ = \text{mho} = \Omega^{-1}$$

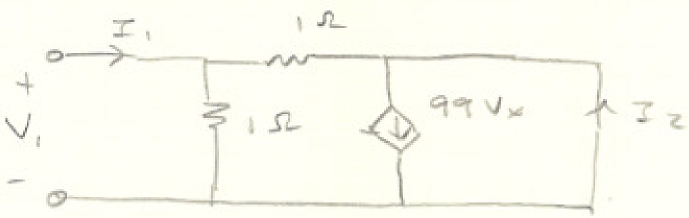
<u>Ex</u>: find the h-parameters given for.



$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

<u>SOL</u>: make $V_2 = 0$



$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \qquad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

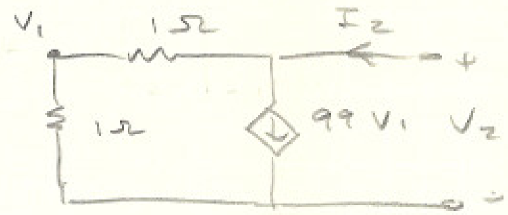
$$I_1 = \frac{V_1}{1} + \frac{V_1}{1} = 2V_1$$

$$\therefore \quad h_{11} = \frac{1}{2} = 0.5\,\Omega$$

$$I_2 = \frac{V_1}{1} - 99V_1 = -98V_1$$

$$\therefore \quad h_{21} = 49$$

for h_{12} & h_{22}, $I_1 = 0$



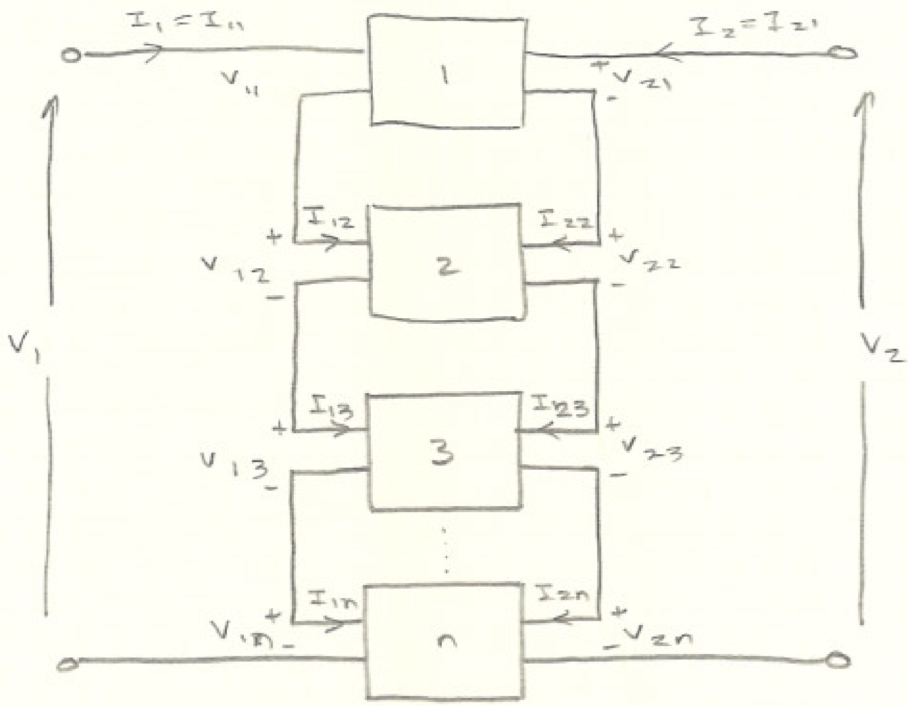
$$I_2 = 99V_1 + \frac{99V_1 - V_1}{1}$$

$$= 197$$

did not finish

ans $\begin{bmatrix} 0.5 & 0.5 \\ 49 & 50 \end{bmatrix}$

THE SERIES CONNECTION.



KVL applies, and in matrix form

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} V_{11} \\ V_{21} \end{bmatrix} + \begin{bmatrix} V_{12} \\ V_{22} \end{bmatrix} + \begin{bmatrix} V_{13} \\ V_{23} \end{bmatrix} + \dots + \begin{bmatrix} V_{1n} \\ V_{2n} \end{bmatrix}$$

as seen

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} I_{12} \\ I_{22} \end{bmatrix} = \begin{bmatrix} I_{13} \\ I_{23} \end{bmatrix} = \begin{bmatrix} I_{1n} \\ I_{2n} \end{bmatrix}$$

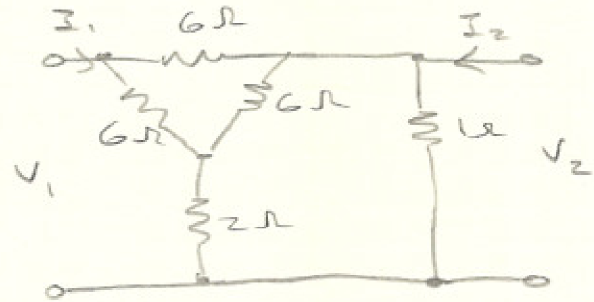
each $\underline{V}$ is related to it's own current vector $\underline{I}$ by the z parameters (ie- $\underline{V} = \underline{Z}\,\underline{I}$) then

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \underline{Z}_1 \begin{bmatrix} I_{11} \\ I_{21} \end{bmatrix} + \underline{Z}_2 \begin{bmatrix} I_{12} \\ I_{22} \end{bmatrix} + \underline{Z}_3 \begin{bmatrix} I_{13} \\ I_{23} \end{bmatrix} + \dots + \underline{Z}_n \begin{bmatrix} I_{1n} \\ I_{2n} \end{bmatrix}$$

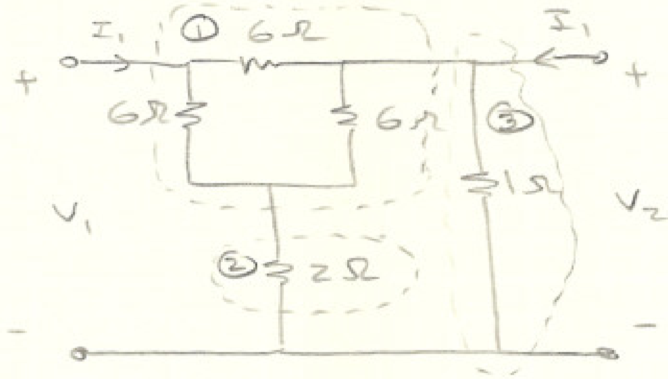
since the vectors are equal, then.

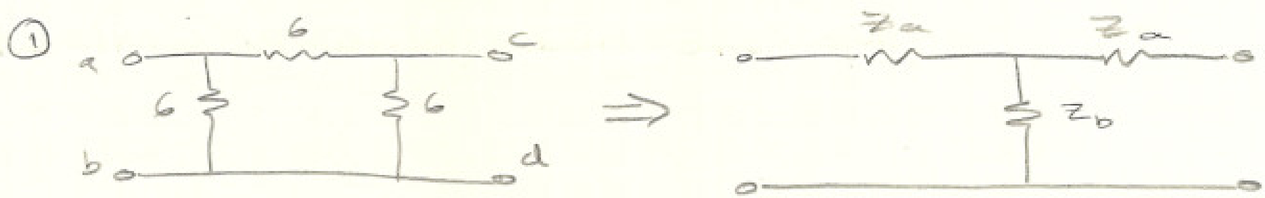
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \left\{ \underline{Z}_1 + \underline{Z}_2 + \underline{Z}_3 + \dots + \underline{Z}_n \right\} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \underline{Z}_{eq} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

EX: reduce the circuit shown to equivalent Pi network.



SOL:





$$Z_{ab} = 6 // (6+6) = 4\ \Omega$$

$$Z_{cd} = 6 // (6+6) = 4\ \Omega$$

so $Z_{11} = Z_{ab} = 4\ \Omega$

$Z_{22} = Z_{cd} = 4\ \Omega$

but $Z_{11} = Z_{ab} = Z_a + Z_b = 4\Omega$

$Z_{22} = Z_{cd} = Z_a + Z_b = 4\Omega$

$\therefore\ Z_b = 2\Omega$

Then we convert Pi network ① to a balance Tee network, the tee elements will be 1/3 those of the Pi network.

$$\therefore \quad \underline{Z} = \begin{bmatrix} 2+2 & 2 \\ 2 & 2+2 \end{bmatrix}$$

the single shunt element of 2 has the T repersentation of

$$\underline{T}_2 = \begin{bmatrix} 1 & 0 \\ 1/2 & 0 \end{bmatrix} \xRightarrow{\text{table}} \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} = \underline{Z}_{eq}(2)$$

although the matrix is singular, it may still be used to form an equivalent Z_{eq} for the series

combinations of the subnetworks 1 & 2

$$\underline{Z}_{eq}(1+2) = \underline{Z}_1 + \underline{Z}_2 = \begin{bmatrix} 6 & 4 \\ 4 & 6 \end{bmatrix}$$

Z_{eq} is not singular, $\Delta Z_{eq} = 20$, from the Table, we convert $\underline{Z}_{eq}$ to $\underline{T}_{12}$

$$\underline{T}_{12} = \begin{bmatrix} \frac{3}{2} & 5 \\ \frac{1}{4} & \frac{3}{2} \end{bmatrix}$$

Combinations of 1 & 2 are in cascade with 3

$$\underline{T}_3 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$\therefore \quad \underline{T}_{eq} = \underline{T}_{12}\,\underline{T}_3 = \begin{bmatrix} \frac{13}{2} & 5 \\ \frac{7}{4} & \frac{3}{2} \end{bmatrix}$$

$\Delta T_{eq} = 1$

from table, we convert $\underline{T_{eq}}$ to $\underline{Y_{eq}}$ to obtain Pi network

$$\underline{Y_{eq}} = \begin{bmatrix} \frac{3}{10} & -\frac{1}{5} \\ -\frac{1}{5} & \frac{13}{10} \end{bmatrix}$$

$\therefore$