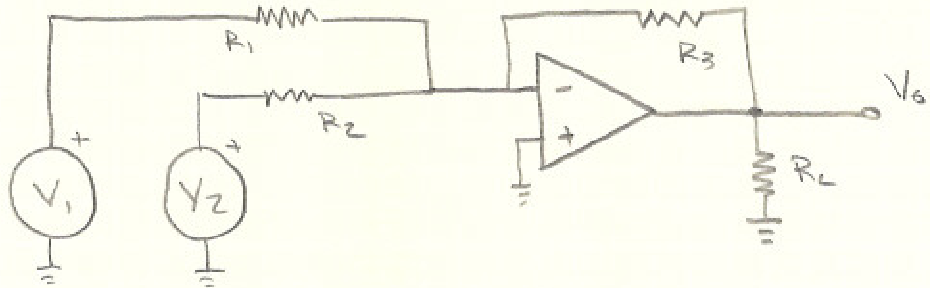
SUMMING AMPLIFIER.

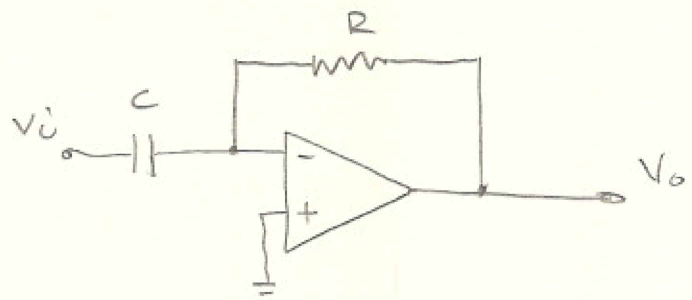


$$V_o = -\frac{R_3}{R_1} V_1 - \frac{R_3}{R_2} V_2$$

if $R_1 = R_2 = R$

$$V_o = -\frac{R_3}{R}(V_1 + V_2)$$

DIFFERENTIAL AMPLIFIER.



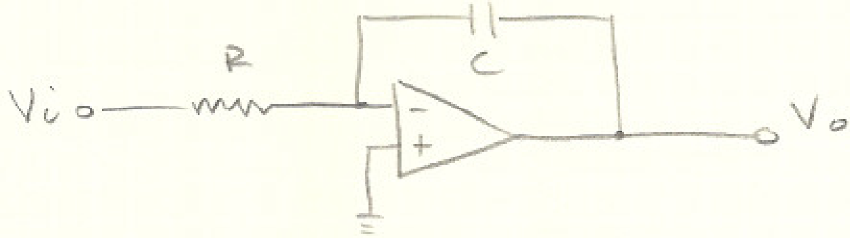
$$V_i = \frac{1}{C}\int i\,dt$$

$$i = C\frac{dv_i}{dt}$$

$$i = -\frac{V_o}{R}$$

$$\therefore V_o = -RC\frac{dv_i}{dt}$$

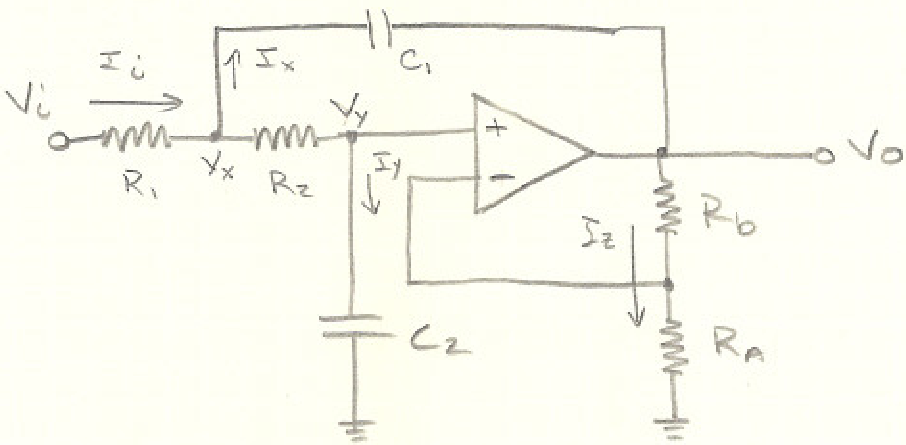
INTEGRATING AMP



$$V_o = -\frac{1}{RC}\int V_i \, dt.$$

KRC FILTER

derive the expression for T(s) given.



$$V_o = (R_b + R_A) I_z$$

$$R_A I_z = V^- \qquad I_z = \frac{V^-}{R_A}$$

$$V_o = (R_b + R_A)\frac{V^-}{R_A}$$

$$V^- = \left(\frac{R_A}{R_A + R_B}\right) V_o \qquad V^- = V^+$$

$$\therefore \quad v^+ = \frac{V_o}{1 + \frac{R_b}{R_A}}$$

let $\quad k = 1 + \frac{R_b}{R_A}$

when $\quad R_a = R_b \Rightarrow k = 2$

$\qquad R_a = 0 \Rightarrow k = \infty$

$\qquad R_b = 0 \Rightarrow k = 1$

using nodal analysis, then

$$I_i = I_x + I_y \qquad (1)$$

$$V_y = v^+ = \frac{V_o}{k}$$

$$I_i = \frac{V_i - V_x}{R_1} \qquad (2)$$

$$I_x = (V_x - V_o)\,\$C_1 \qquad (3)$$

$$I_y = \frac{V_x - V_y}{R_2} = \$C_2 V_y \qquad (4)$$

$$V_x - V_y = \$C_2 R_2 V_y \qquad \text{or}$$

$$V_x = (1 + \$C_2 R_2) V_y \qquad (5)$$

but

$$V_y = \frac{V_o}{k}$$

$$\therefore \quad V_x = (1 + \$C_2 R_2)\frac{V_o}{k}$$

$$\frac{V_i - V_x}{R_1} = (V_x - V_o) s C_1 + \frac{V_x - \frac{V_o}{k}}{R_2}$$

$$\frac{V_i}{R_1} = \frac{V_x}{R_1} + V_x s C_1 - V_o s C_1 + \frac{k V_x - V_o}{k R_2}$$

$$\frac{V_i}{R_1} = V_x \left\{ \frac{1}{R_1} + s C_1 + \frac{1}{R_2} \right\} - \frac{V_o}{k} \left\{ s C_1 k + \frac{1}{R_2} \right\}$$

$$\frac{V_i}{R_1} = V_x \left\{ \frac{R_1 + R_2 + s C_1 R_1 R_2}{R_1 R_2} \right\} - \frac{V_o}{k R_2} \left\{ 1 + s C_1 R_2 k \right\}$$

$$\frac{V_i}{R_1} = (1 + s C_2 R_2) \frac{V_o}{k} \left\{ \frac{R_1 + R_2 + s C_1 R_1 R_2}{R_1 R_2} \right\} - \frac{V_o}{k R_2} \left\{ 1 + s C_1 R_2 k \right\}$$

$$\frac{V_i}{R_1} = \frac{V_o}{k} \left\{ \frac{R_1 + R_2 + s C_1 R_1 R_2 + s C_2 R_1 R_2 + s C_2 R_2^2 + s^2 C_1 C_2 R_1 R_2}{R_1 R_2} - \frac{1}{R_2} \left\{ 1 + s C_1 R_2 k \right\} \right\}$$

$$\frac{V_o}{V_i} = \frac{k / C_1 C_2 R_1 R_2}{s^2 + \left[\frac{(1-k) C_1 R_1 + C_2 R_1 + C_2 R_2}{C_1 C_2 R_1 R_2} \right] s + \frac{1}{C_1 C_2 R_1 R_2}}$$

Sensitivities

$$\omega_o = R_1^{-1/2} C_1^{-1/2} R_2^{-1/2} C_2^{-1/2}$$

$$S_{R_1}^{\omega_o} = S_{R_2}^{\omega_o} = S_{C_1}^{\omega_o} = S_{C_2}^{\omega_o} = -\frac{1}{2}$$

for $\underline{\underline{Q}}$ from $S_x^y = \frac{x}{y}\frac{\partial y}{\partial x}$

$$S_{R_1}^{Q} = -S_{R_2}^{Q} = Q\sqrt{R_2C_2/R_1C_1} - \frac{1}{2}$$

$$S_{C_1}^{Q} = -S_{C_2}^{Q} = Q\left[\sqrt{R_2C_2/R_1C_1} + \sqrt{\frac{R_1C_2}{R_2C_1}}\right] - \frac{1}{2}$$

$$S_{k}^{Q} = Qk\sqrt{R_1C_1/R_2C_2}$$

$$S_{R_a}^{Q} = -S_{R_B}^{Q} = Q(1-k)\sqrt{\frac{R_1C_1}{R_2C_2}}$$

for equal component design, Q sensitivities simplify to

$$S_{R_1}^{Q} = -S_{R_2}^{Q} = Q - \frac{1}{2}$$

$$S_{C_1}^{Q} = -S_{C_2}^{Q} = 2Q - \frac{1}{2}$$

$$S_{k}^{Q} = 3Q - 1$$

$$S_{R_A}^{Q} = -S_{R_B}^{Q} = 1 - 2Q$$

for unity design we have:

$$S_{R_1}^{Q} = -S_{R_1}^{Q} = \frac{1 - \frac{R_1}{R_2}}{2(1 + R_1/R_2)}$$

$$S^{Q}_{C_1} = -S^{Q}_{C_2} = \frac{1}{2}$$

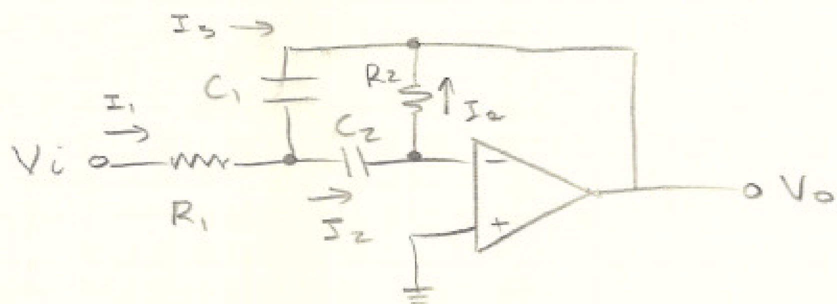
note: since Q sensitivities of the equal component design increase with Q, they may become unexceptable at high Q's.

Another important quality to watch is S^{Q}_{K}, this is of concern at high Q b/c a slight mismatch in R_B/R_A ratio may drive Q to infinity, this will result in oscilating behaviour.

It should be noted that by contrast unity gain designs offer much lower sensitivities.

The designer must carefully weight a number of conflicting factors before choosing a particuler filter design for a given application. These include circuit simplicity, cost, component spread, toneability, and sensitivity

EX: Multiple feedback BP filter.



SENSITIVITY OF $T(j\omega)$

In general:

$$S_x^{T(s)} = \frac{x}{T(s)} \frac{\partial T(s)}{\partial x}$$

let $s = j\omega$, then $T(s) = T(j\omega) = |T(j\omega)| e^{j\theta\omega}$

$$S_x^{T(j\omega)} = \frac{x}{|T(j\omega)| e^{j\theta(\omega)}} \frac{\partial}{\partial x}\left\{|T(j\omega)| e^{j\theta\omega}\right\}$$

$$S_x^{T(j\omega)} = \frac{x}{|T(j\omega)| e^{j\theta(\omega)}} \left\{ e^{j\theta(\omega)} \frac{\partial}{\partial x}\Big[|T(j\omega)|\Big] + j e^{j\theta(\omega)} \frac{\partial \theta(\omega)}{\partial x} |T(j\omega)| \right\}$$

$$S_x^{|T(j\omega)|} = R_e\, S_x^{|T(j\omega)|}$$

$$S_x^{\theta} = \mathrm{Img}\, S_x^{T(j\omega)}$$

Ex: The transfer function of a low pass filter is given by:

$$T(s) = \frac{\omega_o^2}{s^2 + \frac{\omega_o}{Q} s + \omega_o^2}$$

then,

$$S_Q^{T(s)} = \frac{Q}{T} \frac{\partial T}{\partial Q} = \frac{-(\omega_o/Q)s}{s^2 + \frac{\omega_o}{Q} s + \omega_o^2}$$

let $s = j\omega$

$$S_{Q}^{|T(j\omega_0)|} = \mathrm{Re}\, S_{Q}^{T(j\omega)} = -1$$

Ex: Band Pass filter

$$T(s) = \frac{(\omega_0/Q)\, s}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

we are asked to find

$$S_{\omega_0}^{|T(j\omega_0)|} \quad \text{and} \quad S_{Q}^{|T(j\omega_0)|}$$

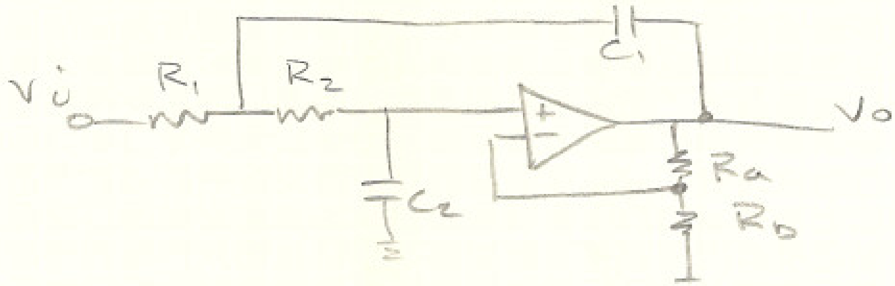
$$S_{\omega_0}^{|T(j\omega_0)|} = -2Q$$

$$S_{Q}^{T(s)} = \frac{s^2 + \omega_0^2}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

$s = j\omega_0$

$$S_{Q}^{T(s)} = 0$$

Ex: Given the low pass filter shown below



$$K = 1 + \frac{R_b}{R_a}$$

$$\frac{V_o}{V_i} = T(s) = \frac{k}{R_1C_1C_2R_2\, s^2 + \{(1-k)R_1C_1 + R_1C_2 + R_2C_2\}s + 1}$$

$$\omega_o = \frac{1}{\sqrt{R_1C_1R_2C_2}}$$

$$Q = \frac{1}{(1-k)\sqrt{R_1C_1/R_2C_2} + \sqrt{\frac{R_1C_2}{R_2C_1}} + \sqrt{\frac{R_2C_2}{R_1C_1}}}$$

Equal component design gives.

$$R_1 = R_2 = R, \quad C_1 = C_2 = C$$

$$\omega_o = \frac{1}{RC} \quad ; \quad Q = \frac{1}{3-k}$$

Design equations are:

$$RC = \frac{1}{\omega_o}, \quad k = 3 - \frac{1}{Q}$$

$$R_b = (k-1)R_a$$

Question: using equal components, specify elements for a 2nd order LP filter with

$$f_o = 1\ \text{kHz} \quad Q = 5$$

Solution: start by choosing standard easily availible capacitor, $C = 10\,nF$

$$R = \frac{1}{\omega_o C} = \frac{1}{2\pi * 10^3 * 10^{-9}} = 15.92\ k\Omega$$

(use $R = 15.8\ k\Omega$, 1%)

$$k = 3 - \frac{1}{Q} = 3 - \frac{1}{5} = 2.8$$

$\frac{R_b}{R_a} = 2.8 - 1$, let us choose $R_a = 10\,k\Omega$, 1% (availible)

$\therefore R_b = 17.8\ k\Omega$, 1%

now let us investigate the effect of a 1% variation of each component in our LP design.

From previous work, we found that:

$$S^{\omega_0}_{R_1} = S^{\omega_0}_{C_1} = S^{\omega_0}_{R_2} = S^{\omega_0}_{C_2} = -\frac{1}{2}$$

hence, a 1% increase in any R_1, R_2, C_1, C_2 causes a 0.5% decrease in ω_0

$$S^{Q}_{R_1} = -S^{Q}_{R_2} = Q - \frac{1}{2}$$

$$S^{Q}_{C_1} = -S^{Q}_{C_2} = 2Q - \frac{1}{2}$$

$$S^{Q}_{K} = 3Q - 1$$

$$S^{Q}_{R_A} = -S^{Q}_{R_b} = 1 - 2Q$$

a 1% i/c in R_1 increases Q by 4.5%. Similairly a 1% capacitive variation results in Q variation by 9.5%

$m = k + \sqrt{k^2+1}$

$k = \frac{n}{2Q^2} - 1$

using unity gain option, design of a LP filter
$f_0 = 10$ kHz $Q = 2$

DESIGN

Pick $C_1 = 1nF$

$4Q^2 = (4)(2)^2 = 16$

choose $n = 20$ since $n \geq 4Q^2$

then $nC = 20nF$

$$k = \frac{n}{2Q^2} - 1 = \frac{20}{2 \cdot 2^2} - 1 = 1.5$$

$$m = k + \sqrt{k^2 - 1} = 2.618$$

$$R = \frac{1}{\sqrt{mn}\, \omega_0 C}$$

$$R = \frac{1}{\sqrt{2.618 \cdot 20}\, 2\pi \cdot 10^4 \cdot 10^{-9}} = 2.199\ k\Omega$$

choose $2.21\ k\Omega$, 1% availible.

$mR = 5.758\ k\Omega$, (use $5.76\ k\Omega$, 1%)

Q// for the above design, investigate the effect of a 1% variation in each component.

SOL// As in the previous example, a 1% increase in any R,C ... cause a 0.5% d/c in ω_0