EX: same circuit shown above, find A, B, C, D, parameters.

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_1$$

SOL: $I_2 = 0$, $\left.\dfrac{V_1}{V_2}\right|_{I_2=0} = A$

$$V_1 = 4I_1 + \frac{1}{s}I_1 = \left(4 + \frac{1}{s}\right)I_1$$

and $V_2 = \frac{1}{s}I_1$, $I_1 = sV_2$

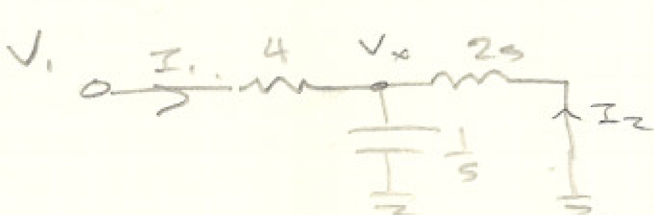
$$V_1 = \left(\frac{4s+1}{s}\right)sV_2$$

$$\therefore \quad A = 4s+1$$

...

$$\left.\frac{I_1}{V_2}\right|_{I_2=0} = C = s$$

$$V_2 = 0, \quad \left.\frac{V_1}{I_2}\right|_{V_2=0} = -B$$



for y_{21} obtain $I_2(s)$ by a current using a circuit in (a)

$$I_1(s) = y_{11} V_1(s)$$

and with careful attention to the direction of I_2, then

$$I_2(s) = \left[\frac{-1/s}{(1/s + 2s)} \right] y_{11} V_1(s)$$

$$\therefore \quad \frac{I_2}{V_1}(s) = \frac{-1}{8s^2+2s+4}$$

likewise

$$y_{22} = \left. \frac{I_2}{V_2} \right|_{V_1=0} = \frac{4s+1}{8s^2+2s+4}$$

$$y_{12} = \left. \frac{I_1}{V_2} \right|_{V_2=0} = -\frac{1}{8s^2+2s+4}$$

$\therefore$

$$\underline{Y}(s) = \begin{bmatrix} \frac{2s^2+1}{8s^2+2s+4} & \frac{-1}{8s^2+2s+4} \\ \frac{-1}{8s^2+2s+4} & \frac{4s+1}{8s^2+2s+4} \end{bmatrix} \quad S$$

TEXT (OPTIONAL) "PASSIVE & ACTIVE ANALYSIS & SYNTHESIS" BY ADAM BUDAK, WAVELAND PRESS INC.

TOPICS.

- OPERATION CALCULUS TO NETWORK ANALYSIS, NETWORK THEOREMS.
- FUNDEMENTALS OF POLES AND ZEROS IN THE COMPLEX FREQUENCY PLANE. THE EFFECT OF POLES/ZEROS ON NETWORK RESPONCE
- ANALYSIS OF 2 PORT NETWORKS, Z PARAMETERS Y PARAMETERS, ABCD PARAMETERS, RECIPOCITY THEOREM, CONCEPT OF SYMETRY, SERIES PARALLEL AND CASCADE INTERCONECTIONS OF THE 2 PORT NETWORK.
- SYNTHESIS OF FOSTER AND CAUER INPUT IMPEDANCIES FOR RC AND LC NETWORKS
- PROPERTIES OF 2^{ND} ORDER SYSTEMS, INCLUDING ROOT LOCI AND SENSITIVITY FUNCTIONS
- STUDY OF THE OP AMP, AND OP AMP CIRCUIT ANALYSIS,

MARK BREAKDOWN.

- MIDTERM 35%
- FINAL 65%

LAPLACE TRANSFORM

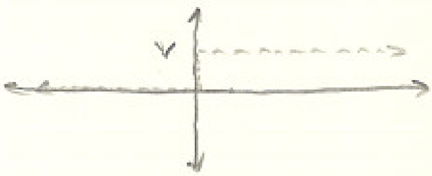
the laplace transform of a function f(t) is defined as.

$$\mathcal{L} f(t) = \int_0^\infty e^{-st} f(t)\,dt = F(s)$$

$$\mathcal{L}^{-1} F(s) = f(t)$$

$$\mathcal{L} f'(t) = sF(s) - f(0)$$

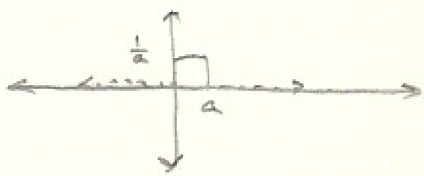
recall step function.



$$u(t) = \begin{cases} 0 & t < 0 \\ V & t \geq 0 \end{cases}$$

$$\mathcal{L}\, u(t) = \int_0^\infty e^{-st}\, u(t)\, dt = \frac{V}{s}$$

recall unit pulse.



$$\delta(t) = \begin{cases} 0 & t < 0 \\ 1/a & 0 < t < a \\ 0 & t > a \end{cases}$$

EX.

$$F(s) = \frac{1 - e^{-as}}{as} \qquad \text{when} \quad a \to 0$$

if we directly substitude we get $\frac{0}{0}$, ∴ we use l'Hopitals rule, which states if

$$f_1(t) = 0 \quad f_2(t) = 0 \quad t = a$$

then

$$\lim_{t \to a} \frac{f_1(t)}{f_2(t)} = \lim_{t \to a} \frac{f_1'(t)}{f_2'(t)}$$

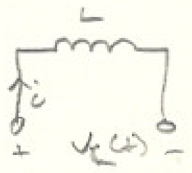
$$\therefore \quad F(s) = \frac{a e^{-as}}{a} = 1$$

recall convolution.

$$\mathcal{L}^{-1}\, F(s) \cdot G(s) = \int_0^t f(\tau)\, g(t - \tau)\, d\tau$$

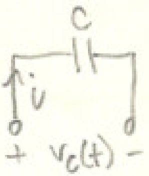
TIME DOMAIN.

induction.



$$\boxed{V_L(t) = L\frac{di}{dt}}$$

capacidence

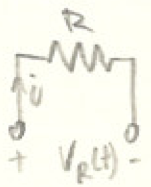


$$q_c = C v_c$$

$$\frac{dq_c}{dt} = C\frac{dv_c}{dt}$$

$$\boxed{i(t) = C\frac{dv_c}{dt}}$$

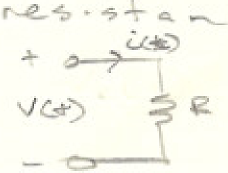
resistance



$$\boxed{V_R(t) = R\,i(t)}$$

IMPEDANCE FUNCTIONS AND EQUIVALENT SOURCE REPRESENTATION OF INITIAL COND

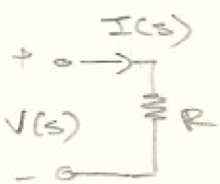
resistance



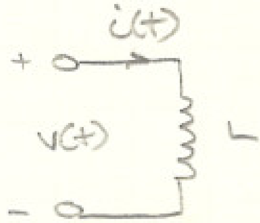
$$V_R(t) = i(t)\,R.$$

$$\mathcal{L}\, V_R(t) = R\,\mathcal{L}\, i(t)$$

$$V(s) = R\,I(s)$$



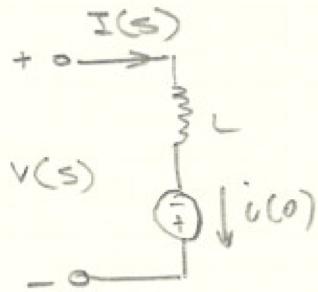
inductance



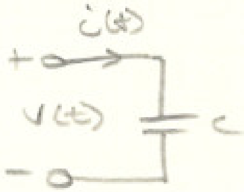
$$v(t) = L\frac{di(t)}{dt}$$

$\mathcal{L}$ both sides

$$V(s) = L\left(sI(s) - i(0)\right)$$



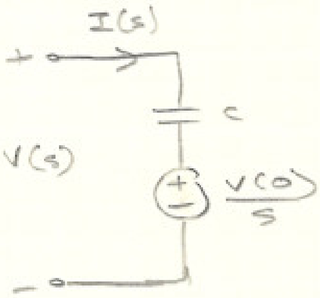
capacidence



$$i(t) = C\frac{dv}{dt}$$

$\mathcal{L}$ both sides, we get

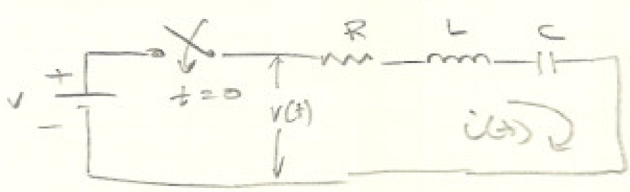
$$I(s) = C\left[sV(s) - v(0)\right]$$



rearrange

$$V(s) = \frac{I(s)}{sC} + \frac{v(0)}{s}$$

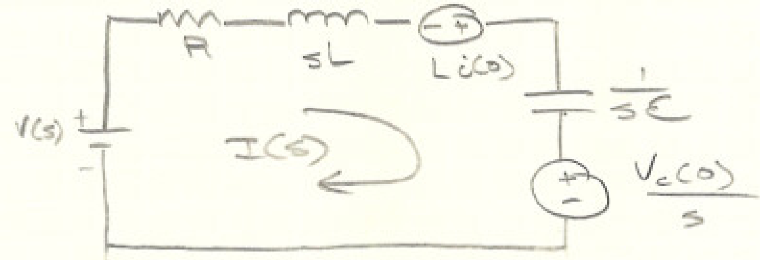
Ex transient responce of an RLC series circuit



at $t=0$, assume that $L = 10$ H, $V = 10$ V, $R = 100\,\Omega$

$C = 1000\,\mu F$, $i_L(0) = 1\,A$ and $v_c(0) = 5\,V$

TRANSFORMED CIRCUIT.



$$\therefore \quad \frac{V}{s} = \left[R + sL + \frac{1}{sC}\right] I(s) - L\,i(0) + \frac{V_c(0)}{s}$$

or
$$I(s) = \left[\frac{V}{s} + L\,i(0) - \frac{V_c(0)}{s}\right] \cdot \left[\frac{s/L}{s^2 + R/L\,s + \frac{1}{LC}}\right]$$

substitute values.

$$I(s) = \left[\frac{10}{s} + 10 - \frac{5}{s}\right] \cdot \left[\frac{s/10}{s^2 + 10s + 100}\right]$$

after simplifing and completing the square

$$I(s) = \frac{1/2}{(s+5)^2 + (\sqrt{75})^2} + \frac{s + 5 - 5}{(s+5)^2 + (\sqrt{75})^2}$$

$\mathcal{L}^{-1}$

$$i(t) = -\frac{4.5}{\sqrt{75}} e^{-5t} \sin\sqrt{75}\,t + e^{-5t}\cos\sqrt{75}\,t$$

let $\omega = \sqrt{75}$

$$i(t) = e^{-5t}\left[-\frac{4.5}{\omega}\sin\omega t + \cos\omega t\right]$$

$$A\sin(\omega t + \phi) = A\sin\omega t\cos\phi + A\sin\phi\cos\omega t$$

Comparing coeffients.

?

$$A\cos\phi = \frac{-4.5}{\omega}$$

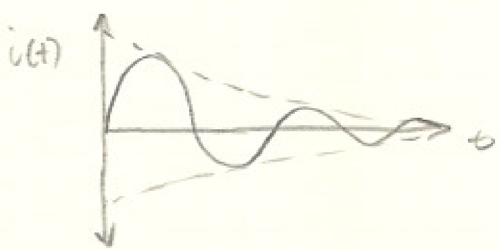
$$A\sin\phi = 1$$

$$\therefore \quad \tan\phi = \frac{1}{\frac{-4.5}{\omega}} \qquad \phi = 1.9245 \text{ rad.} = 117.46^\circ$$

$$\& \quad A = \sqrt{\left(\frac{-4.5}{\omega}\right)^2 + (1)^2} = 1.12694$$

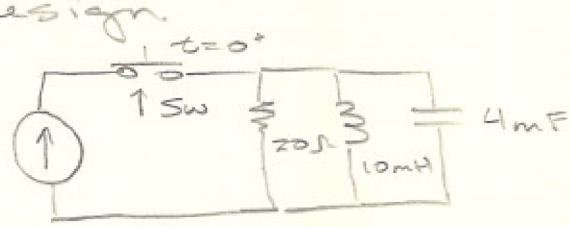
$$\therefore \quad i(t) = 1.12694\, e^{-5t} \sin(\sqrt{75}t + 117.45) \quad (A)$$

this can be checked by setting $t=0$, we can see that we achieved our initial current.



(decaying transiant state)

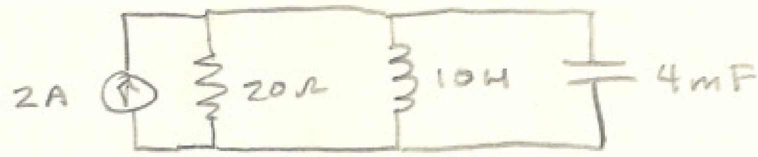
Ex. a parrellel RLC circuit is used in communications network and in filter design.



sw has been closed for a long time at $t=0^+$, sw is opened, find $v(t)$ for $t>0$

continuing from last day.

SOL: at $t < 0$

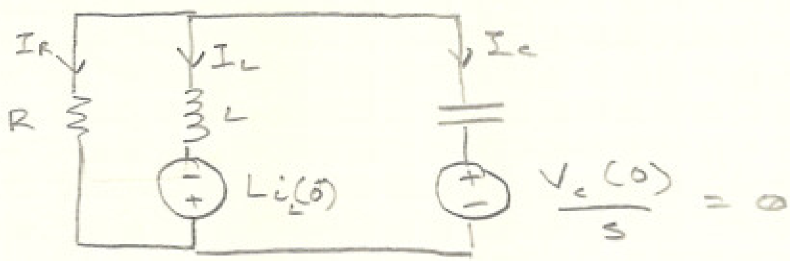


initial conditions are:

$$i_L(0^-) = i_L(0^+) = 2A$$

$$v_c(0^-) = v_c(0^+) = 0V$$

at $t \geq 0$ we have transformed cir.



we know that

$$I_R + I_L + I_c = 0 \qquad \text{A}$$

$$I_L = \frac{V(s) + L i_L(0^+)}{sL}$$

which is the same as

$$V(s) = sLI_L - Li_L(0)$$

hence equation A can be written as

$$\frac{V(s)}{R} + \frac{V(s) + Li_L(0)}{sL} + sCV(s) = 0$$

$$V(s)\left[\frac{1}{R} + \frac{1}{sL} + sc\right] = -\frac{Li_L(0)}{sL}$$

$$i_L(0^+) = i_L(0^-) = 2A$$

$$V(s)\left[\frac{s^2RLC + sL + R}{sRL}\right] = \frac{-2}{s}$$

substituting we get.

$$V(s) = \frac{(-2)(20 \cdot 10)}{(800 \cdot 10^{-3})s^2 + 10s + 20}$$

$$V(s) = \frac{-400}{0.8s^2 + 10s + 20}$$

$$V(s) = \frac{-400}{0.8(s^2 + 12.5s + 25)}$$

$$V(s) = \frac{-500}{(s+10)(s+2.5)}$$

partial fractions.

$$\frac{A}{s+10} + \frac{B}{s+2.5}$$

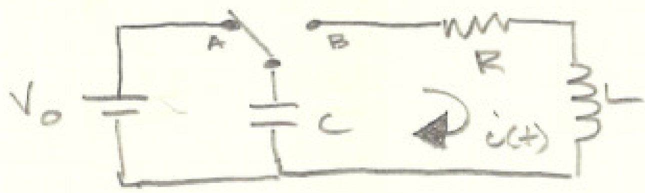
$$V(s) = \frac{66.67}{(s+10)} - \frac{66.67}{(s+2.5)}$$

$\therefore \quad v(t) = 66.67\left[e^{-10t} - e^{-2.5t}\right]$

for $t > 0$

check for $t = 0$ . . .

Ex. given the following circuit.



the switch as been in position A for a long time, AT $t=0$ the swith is put in the B position.

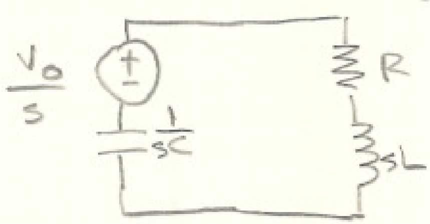
find $i(t)$ for $t > 0$

SOL: initial conditions

$V_c = V_0$

$i_L(0^+) = 0$

hence the transformed circuit looks like



note: no current source by the inducter b/c of initial conditions

the loop equation.

$$\frac{V_0}{s} = \left[\frac{1}{sc} + R + sL\right] I(s)$$

$$I(s) = \frac{V_0}{L}\left[\frac{1}{s^2 + \frac{R}{L}s + \frac{1}{LC}}\right]$$

$$I(s) = \frac{V_0}{L}\left[\frac{1}{\left(s+\frac{R}{2L}\right)^2+\left(\frac{1}{LC}-\frac{R^2}{4L^2}\right)}\right]$$

let $a = \frac{R}{2L}$ and $\omega^2 = \left(\frac{1}{LC} - \frac{R^2}{4L^2}\right)$

$$\therefore \quad I(s) = \frac{V_0}{L}\left[\frac{1}{(s+a)^2+\omega^2}\right]$$

$$I(s) = \frac{V_0}{\omega L}\left[\frac{\omega}{(s+a)^2+\omega^2}\right]$$

$\mathcal{L}^{-1}$

$$i(t) = \frac{V_0}{\omega L} e^{-at} \sin\omega t \quad \text{for } t > 0$$

observations: if $R=0$ the $a=0$ and ω becomes $1/\sqrt{LC}$ and we have a harmonic function that does not decay.

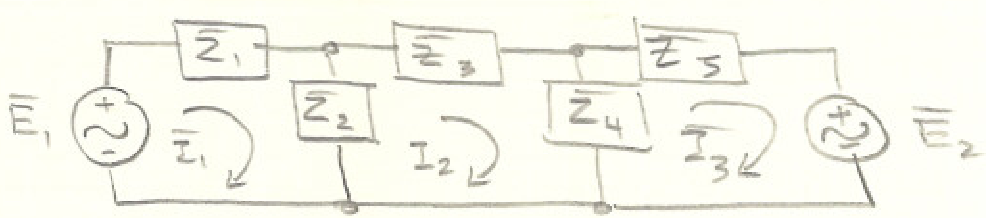
when $\omega^2 > 0$, we get an overdemped responce.

when $\omega^2 = 0$, $R^2 = \frac{4L}{C}$, critical responce happens and there are no ossilations (domped)

when $\omega^2 < 0$, we get a demped responce

NETWORK THEOREM.

LOOP/MESH ANALYSIS



LOOP 1

$$\bar{E}_1 = (\bar{Z}_1 + \bar{Z}_2)\bar{I}_1 - \bar{Z}_3\bar{I}_2 - \emptyset\bar{I}_3$$

LOOP 2

$$0 = -\bar{Z}_2\bar{I}_1 + (\bar{Z}_2 + \bar{Z}_3 + \bar{Z}_4)\bar{I}_2 - \bar{Z}_4\bar{I}_3$$

LOOP 3

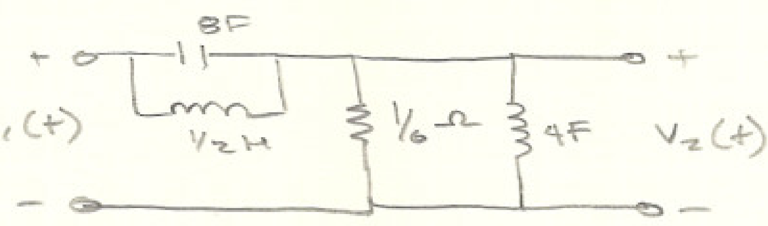
$$-\bar{E}_2 = \emptyset\bar{I}_1 - \bar{Z}_4\bar{I}_2 + (\bar{Z}_4 + \bar{Z}_5)\bar{I}_3$$

we can write the following in matrix form

$$\bar{\bar{E}} = \bar{\bar{Z}} \cdot \bar{\bar{I}}$$

$$\begin{bmatrix} \bar{E}_1 \\ \bar{0} \\ -\bar{E}_2 \end{bmatrix} = \begin{bmatrix} (\bar{Z}_1 + \bar{Z}_2) & -\bar{Z}_2 & 0 \\ -\bar{Z}_2 & (\bar{Z}_2 + \bar{Z}_3 + \bar{Z}_4) & -Z_4 \\ \emptyset & -Z_4 & (\bar{Z}_4 + \bar{Z}_5) \end{bmatrix} \cdot \begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \\ \bar{I}_3 \end{bmatrix}$$

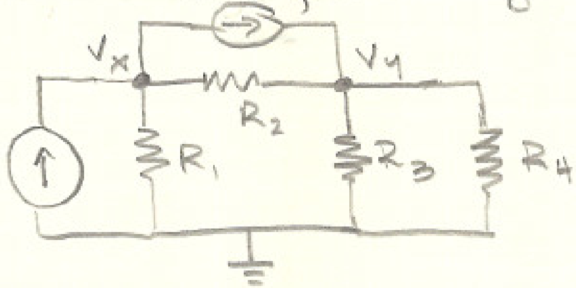
EX. determine the DE of

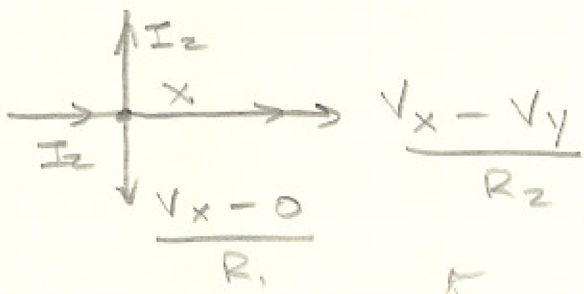


assume that $v_1(t)$ is arbitrary, also assume that there is no energy in the caps/inductors.

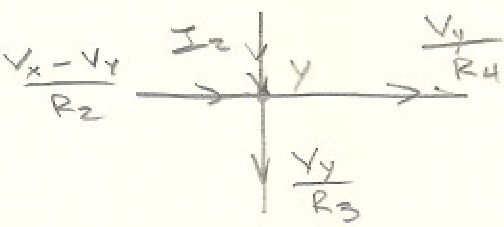
NODAL ANALYSIS.

Consider the following:





these are currents.



what we are saying is that all the current in is equal to all the current out. for each node.

NODE X

$$I_1 - I_2 = V_x\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - V_y\left(\frac{1}{R_2}\right)$$

NODE Y

$$I_2 = -V_x\left(\frac{1}{R_2}\right) + V_y\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right)$$

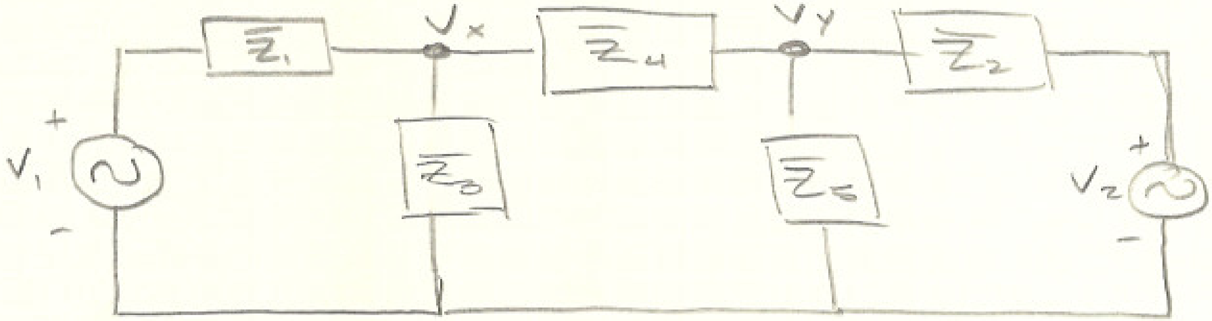
now we can create a matrix

$$\underline{I} = \underline{Y}\,\underline{V}$$

inverted resistance.
(admitance).

$$\begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} 1/R_1 + 1/R_2 & -1/R_2 \\ -1/R_2 & 1/R_2 + 1/R_3 + 1/R_4 \end{bmatrix} \begin{bmatrix} V_x \\ V_y \end{bmatrix}$$

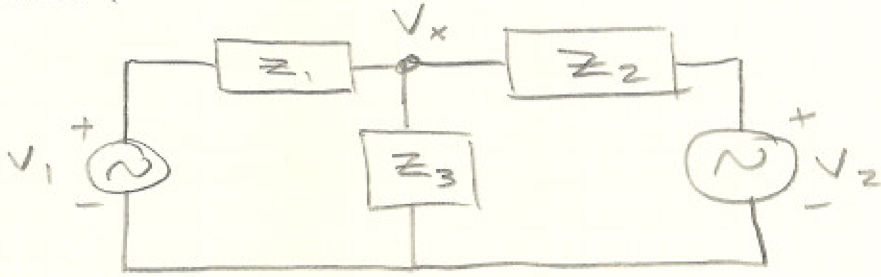
Ex.



$$\begin{bmatrix} \dfrac{V_1}{Z_1} \\ \dfrac{V_2}{Z_2} \end{bmatrix} = \begin{bmatrix} \dfrac{1}{Z_1} + \dfrac{1}{Z_3} + \dfrac{1}{Z_4} & -\dfrac{1}{Z_4} \\ -\dfrac{1}{Z_4} & \dfrac{1}{Z_2} + \dfrac{1}{Z_4} + \dfrac{1}{Z_5} \end{bmatrix} \begin{bmatrix} V_x \\ V_y \end{bmatrix}$$

THEVENINS' THEOREM

consider.

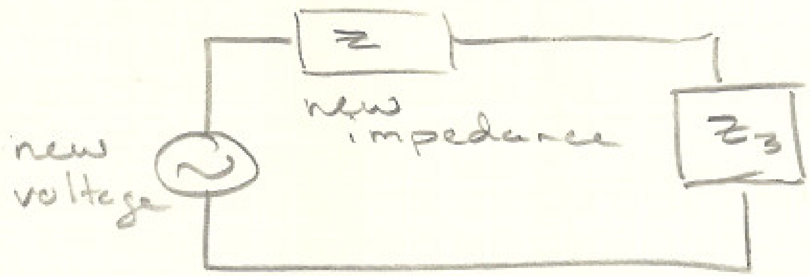


using nodal analysis.

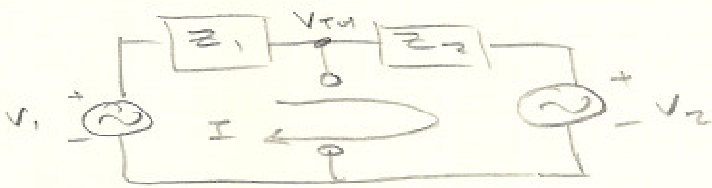
$$\frac{V_1}{Z_1} + \frac{V_2}{Z_2} = V_x\left[\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}\right]$$

using thevenin's

if we were to open Z_3 and find the voltage across the nodes, then short the sources and find the resistance we can use those to find the current in Z_3



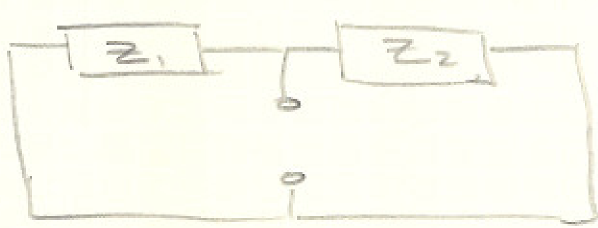
from the last problem



$$V_1 + V_2 = (Z_1 + Z_2)I$$

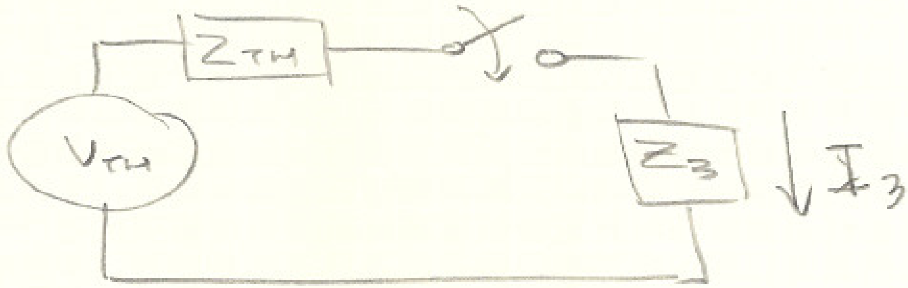
$$I = \frac{V_1 + V_2}{Z_1 + Z_2}$$

$$V_1 = Z_1 I + V_{TH} \implies V_{TH} = V_1 - Z_1 I$$



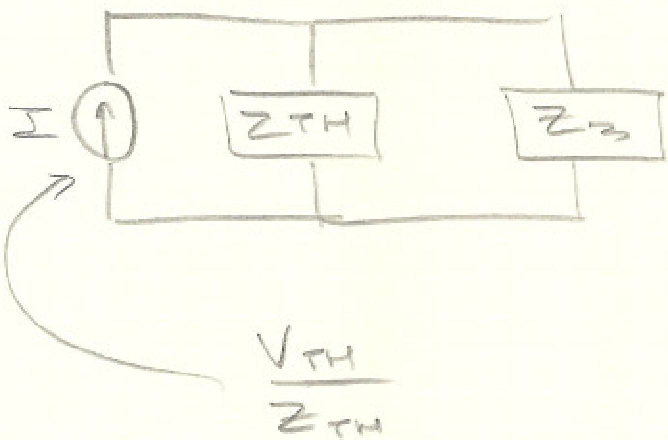
$$Z_{TH} = Z_1 // Z_2$$

then we have



$$I_3 = \frac{V_{TH}}{Z_{TH} + Z_3}$$

NORTONS THEOREM



CRAMER'S RULE

consider 2 equations.

$$a_{11}x_1 + a_{12}x_2 = b_1$$
$$a_{21}x_1 + a_{22}x_2 = b_2$$

this can be rearranged to state.

$$x_1 = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

$$x_2 = \frac{a_{11}b_2 - a_{21}b_1}{a_{11}a_{22} - a_{12}a_{21}}$$

we may rearrange from the above equations.

$$x_1 = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

$$x_2 = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

In general for n linear equations

$$\underline{A}\,\underline{x} = \underline{b}$$

then $\quad x_1 = \dfrac{\det A_1}{\det A} \quad x_n = \dfrac{\det A_n}{\det A_1}$

EX. Solve by Cramers rule.

$$3x_1 + 2x_2 + x_3 = 7$$
$$x_1 - x_2 + 3x_3 = 3$$
$$5x_1 + 4x_2 - 2x_3 = 1$$

SOL:

$$\det A = \begin{vmatrix} 3 & 2 & 1 \\ 1 & -1 & 3 \\ 5 & 4 & -2 \end{vmatrix} = 13$$

$$\det A_1 = \begin{vmatrix} 7 & 2 & 1 \\ 3 & -1 & 3 \\ 1 & 4 & -2 \end{vmatrix} = -39.$$

$$\det A_2 = \begin{vmatrix} 3 & 7 & 1 \\ 1 & 3 & 3 \\ 5 & 1 & -2 \end{vmatrix} = 78$$

$$\det A_3 = \begin{vmatrix} 3 & 2 & 7 \\ 1 & -1 & 3 \\ 5 & 4 & 1 \end{vmatrix} = 62$$

$$x_1 = \frac{\det A_1}{\det A} = -3$$

$$x_2 = \frac{\det A_2}{\det A} = 6$$

$$x_3 = \frac{\det A_3}{\det A} = 4$$

EX... continued from the other day.

from last day we had the DE.

$$12\frac{d^2v_2}{dt^2} + 6\frac{dv_2}{dt} + 2v_2 = 8\frac{d^2v_1}{dt^2} + 2v_1$$

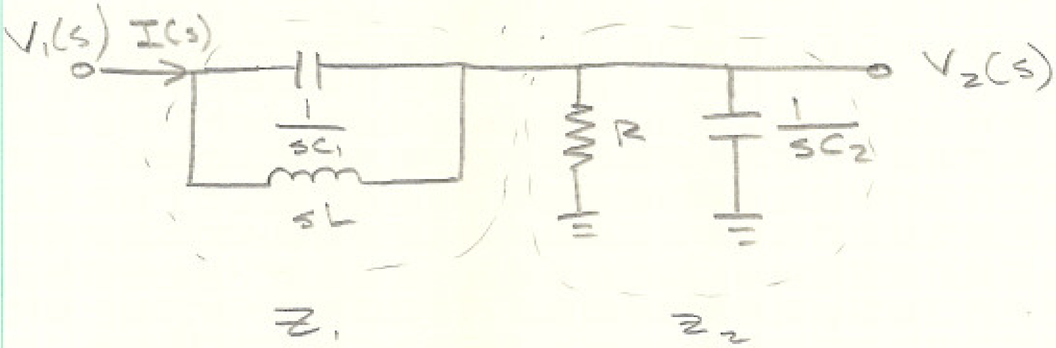
assuming the system is relaxed, (all initial conditions are 0), then we can find the transfer function (TF), we take the laplace transform of both sides.

$$\left[12s^2 + 6s + 2\right]V_2 = \left[8s^2 + 2\right]V_1$$

$$\frac{V_2(s)}{V_1(s)} = G(s) = \frac{8s^2+2}{12s^2+6s+2}$$

If $V_1(t)$ is known, we can find $V_2(s)$ and $V_2(t)$

<u>note</u>: we may transform the circuit.



$$Z_1 = \frac{(sL)(1/sC_1)}{(sL + 1/sC_1)} \qquad Z_2 = \frac{R(\frac{1}{sC_2})}{R + 1/sC_2}$$

$$V_1(s) = (Z_1 + Z_2)I(s)$$

$$I(s) = \frac{V_2(s)}{Z_2}$$

$$\therefore \frac{V_2(s)}{V_1(s)} = \frac{Z_2}{Z_2 + Z_1}$$

and we get the same result solving in this method.