The Cramer Rao Lower Bound of Magnetic Resonance Phase Image Acquisitions: Comparison with Bayesian Constrained Reconstruction

MacDonald ME, Swailies N, Smith MR, Nielsen JN, Frayne R

Biomedical Engineering, Mechanical Engineering, Electrical Engineering, Radiology, Clinical Neuroscience, and the Hotchkiss Brain Institute, at the University of Calgary;

Seaman Family Magnetic Resonance Research Centre, Foothills Medical Centre, Alberta Health Services; Calgary, Canada

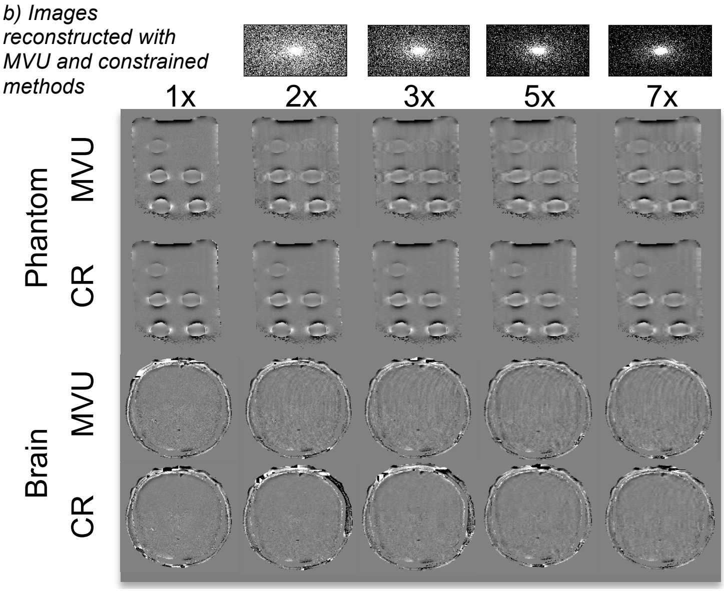
Introduction: The Cramer Rao Lower Bound (CRLB) represents the smallest variance achievable for an unbiased estimator. The estimator that meets the CRLB is called the minimum variance unbiased (MVU) estimator. When reconstructing magnitude images from magnetic resonance (MR) *k*-space, the MVU image estimator is found to be the Fourier transform in the fully sampled case [1]. By using a Bayesian approach, *i.e.*, a biased reconstruction, such as many of the proposed constrained sensing reconstructions [2-4] then a lower image mean square error (MSE) may be achievable over a certain range of parameters [5]. Several MR imaging methods require phase images, including: phase contrast velocity encoding, susceptibility-weighted imaging, B0-field mapping and quantitative susceptibility mapping. The probability density function for noise in phase images is known to be quite different than in magnitude images. It is our hypothesis that the Bayesian constrained reconstruction will perform better at producing phase images than the MVU over a range of acceleration factors. In this work we perform numeric simulations on real data, obtained by imaging a custom imaging phantom and a human brain, to demonstrate variation in the image phase MSE with respect to acceleration factor.

Methods: An imaging phantom was constructed from agarose doped with 0.45% NaCl to match the conductivity properties of the brain. The phantom had six cylindrical cores, which were filled with a similar solution of agarose material and 0.0%, 0.1%, 0.2%, 0.3%, 0.4%, and 0.5% gadolinium contrast agent by concentration (Magnevist, Bayer Healthcare Pharmaceuticals). The regions doped with contrast are intended to create a phase offset in acquired images due to magnetic susceptibility difference [3,6]. Imaging of the phantom and *in vivo* subject were performed with a 3 T MR scanner (Discovery 750, GE Healthcare). A 3D spoiled gradient recalled echo sequence was used for imaging with 1 mm^3 isotropic resolution, with TR and flip angle of 5 ms and 10 degrees, respectively. Raw data was acquired with the vendor-supplied raw data server feature and used as an input for the numerical simulation. Reconstruction was performed with two strategies: 1) with zero filling and use of the FFT, which represents the image MVU estimator, and 2) with an L1 norm constrained wavelet reconstruction, which represents a Bayesian reconstruction. The *k*-space data was under sampled with the sampling distributions shown in Fig 1a. The phase was unwrapped and the background field was removed. The fully sampled MVU estimator was considered the gold standard image, and the MSE measures were recorded for all reconstructions.

Results: Fig 1b shows example phase images at multiple acceleration factors for both reconstruction types. Fig 2 shows a plot of the MSE as a function of the acceleration factor. The constrained wavelet reconstruction (Bayesian reconstruction) was able to achieve lower phase MSE than the MVU for most acceleration factors.

Discussion: Using Bayesian-constrained reconstruction techniques has become very popular for some modern MR imaging applications. Previous simulations have demonstrated that MSE values in magnitude images are reduced when using constrained reconstructions [1,2,7]. This study demonstrates how the phase images can also have lower MSE under certain conditions, *i.e.*, over a range of acceleration levels.

Fig 1: a) Sampling b) Images reconstructed with MVU and constrained methods



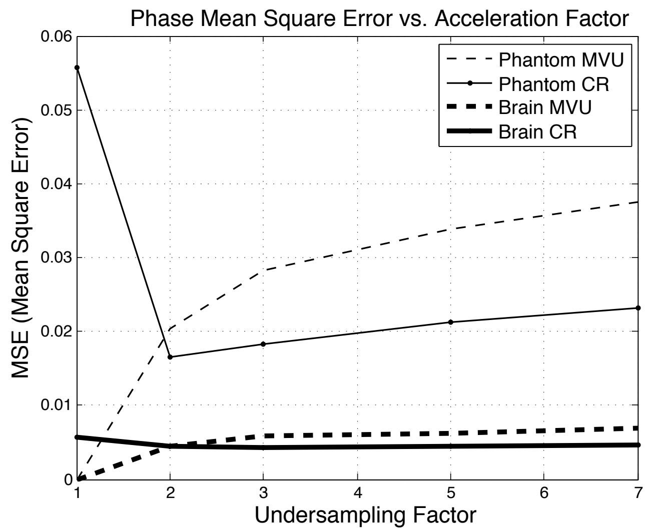


Fig 2: MSE vs Acceleration Factor. Data includes both phantom and in vivo measurements for a region measured inside the object..

References

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